44. 191.

Clarke, Henry?

A

SYSTEM OF ARITHMETIC,

WITH THE

PRINCIPLES OF LOGARITHMS;

COMPILED FOR

MERCHANT-TAYLORS' SCHOOL.

THIRD EDITION.

PREFACE.

IT is one object of the present Work to abridge the labour of teaching and learning Arithmetic, by rendering the Rules as simple, and the explanations as intelligible, as possible.

To effect this, several Rules, as those for *Interest, Gain and Loss, Exchange,* &c., are briefly adverted to, or omitted, because they are but varied applications of the RULE OF THREE; and because, in Counting Houses, TABLES OF INTEREST AND EXCHANGE are always consulted.

Some other Rules, as those for *Permutations, Combinations,* &c., are likewise omitted, because answers can be much more readily obtained by *Algebra.*

The Rules for the use of *Logarithms* are added, as affording a clearer method of extracting the *Cube,* or any higher, *Root,* than the ordinary arithmetical one.

It is a second object, to provide the most useful Rules only, previous to the commencement of *Algebra;* for this is now very generally taught, and is a subject, of which no boy ought to be permitted to remain ignorant.

R. F. CLARKE.

August, 1844.

CONTENTS.

Application of the Rule of Three, or Simple Proportion, viz.

Explanation of the Characters used in this Work.

———————

= *Equal to.* The sign of Equality; as 4 farthings = 1 penny, that is, 4 farthings are equal to 1 penny.

+ *Plus,* or *more.* The sign of Addition; as 3 + 2 = 5, that is 3 added to 2, = 5.

× *Into.* The sign of Multiplication; as 5 × 4 = 20, that is, 5 multiplied into 4 = 20.

÷ *By.* The sign of Division; as 6 ÷ 2 = 3, that is, 6 divided by 2 = 3.

: *Is to* ⎱ The signs of Proportion; as 2 : 4 : : 8 : 16, that is, as 2
: : *So is* ⎰ is to 4, so is 8 to 16.

$\dfrac{3}{\cancel{18}}$ ⎰ The sign of Contraction, or cancelling, which implies that the
$\dfrac{\cancel{12}}{2}$ ⎱ numbers 18 and 12 have been struck out, and the numbers 3 and 2 are to be used instead.

ARITHMETIC.

ARITHMETIC is the Science of computing by Numbers.

Its leading Rules are NUMERATION, ADDITION, SUB-TRACTION, MULTIPLICATION, and DIVISION.

NUMERATION

Teaches to read the value of numbers:

Thus, 23 is read, Twenty-three.

756 is read, Seven hundred and fifty-six.

905 is read, Nine hundred and five.

EXERCISES FOR THE LEARNER.

Write in words the following, 42—87—153—604.

When there are more than three figures, they are nume-rated thus:

2,340 Two thousand, three hundred and forty.

50,129 Fifty thousand, one hundred and twenty-nine.

718,406 Seven hundred and eighteen thousand, four hundred and six.

EXERCISES.

Write in Words 1,325—72,804—291,570—806,093.

When there are more than six figures, they are numerated thus:—

3,209,186 Three millions; two hundred and nine thousand, one hundred and eighty-six.

54,072,831 Fifty-four millions; and seventy two thousand, eight hundred and thirty-one.

908,375,200 Nine hundred and eight millions; three hundred and seventy-five thousand, two hundred.

B

The Table.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
	9	8	7 ;	6	5	4 ;	3	2	1

Note. The figures between each stop are read as if they stood alone; those *before* a comma being always thousands, and those *after* the comma being millions, billions, trillions, &c., according to their place:

Thus, in the Table 987;654,321 are read, 987 millions; 654 thousands, 321.

EXERCISES.

Write in words 1;285,362—40;237,615—327;805,463—901;004,350.

In common calculations more than nine figures are seldom required. But the Table may be extended to any length; as millions, billions, trillions, quadrillions, quintillions, &c.

Thus, 32;407,280;375,049 are read, Thirty-two billions; four hundred and seven thousand, two hundred and eighty millions; three hundred and seventy-five thousand, and forty-nine.

NOTATION

Teaches to write or express the value of numbers:—

Thus, Forty-seven is written........ 47
Two hundred and thirty-five ..235

EXERCISES.

Write in figures the following, Fifty-four — Eighty-nine — Two hundred and thirty-six.

When the number contains thousands—

Thus—Five thousand, two hundred and forty, is written.... 5,240
Eighty-four thousand, and thirty-seven............ 84,037
Six hundred and ninety-one thousand, two hundred and seventy-five............................. 691,275

EXERCISES.

Express in figures, Three thousand, one hundred and eighty-four.
Forty-nine thousand, two hundred and eight.
Six hundred and seventy-three thousand, one hundred and seventy-nine.

When the number contains millions—

Thus—Five millions; two hundred and seventeen thou-
sand, six hundred and forty-five............ 5;217,645
Seventy-nine millions; four hundred and seven
thousand, two hundred and eighty-one...... 79;407,281
Three hundred millions; nine hundred and forty-
five thousand, and eighteen............... 300;945,018

EXERCISES.

Write in figures, Eight millions; two hundred and seventy-one thousand, three hundred and eighty-six.

Nineteen millions; four hundred and five thousand, six hundred and sixty-two.

Three hundred and seven millions; and fifty-nine thousand, one hundred and eighty-seven.

The Romans and other ancient nations, to whom the present method of Notation was unknown, made use of seven capital letters to express any number required; namely, I for one; V for five; X for ten; L for fifty; C for one hundred; D for five hundred; M for one thousand.

XXV expressed Twenty-five;—CVIII One hundred and eight;—CCLXI Two hundred and sixty-one;—MDCCCXLIV One thousand eight hundred and forty-four.

SIMPLE ADDITION

Teaches to add several numbers of the same kind into one total.

Example.

425	Begin at the lowest figure of the right hand column,
214	that is with 3, and add thus, 3 and 4 are 7; and 7 are
561	14; and 1 are 15; and 4 are 19; and 5 are 24. Put
27	down 4 under 3, and add 2 to the next column. Again,
204	2 added to 5 are 7; and 2 are 9; and 6 are 15; and
53	1 are 16; and 2 are 18. Put down 8 under 5, and
——	add 1 to the next column. Again, 1 added to 2 are 3;
Total 1484	and 5 are 8; and 2 are 10; and 4 are 14. Put down 4
====	under 2, and 1 to the left. The total is 1484.

Method of Proof. Begin at the upper instead of the under figure of the right-hand column, and proceed towards the bottom; and if the total is the same, you may conclude the work is right.

B 2

EXERCISES.

[1]	[2]	[3]	[4]	[5]	[6]
23	45	120	458	4790	7583
35	54	314	96	2868	249
12	63	33	342	473	1637
43	20	142	175	3952	104
34	35	320	304	5167	73
21	42	214	28	4285	3246

[7]	[8]	[9]	[10]
21876	40976	50728	147628
3497	31268	62975	705293
854	91345	4386	4316
46712	64071	73241	265782
3145	21768	785	20147
72608	39125	54763	381594

11. Add together, 21 ;—34 ;—45 ;—13 ;—24, and 15.

12. Add together, 325 ;—406 ;—214 ;—131 ;—523, and 31.

13. Add together, Fifty-seven ;—ninety-six ; seventy-five ;—eighty-one ;—thirty-six ; and sixty seven.

14. Add together, 763 ;—849 ;—217 ;—340 ;—285, and 136.

15. Add together, 7184 ;—2367 ;—1053 ;—298 ; 3764, and 5470.

16. Add together, Three thousand, two hundred and ninety-five ;—one hundred and eighty-seven ;—five thousand, four hundred and thirty-eight ;—three thousand, seven hundred and twenty-nine ;— one hundred and fifty-four ;—eight thousand, three hundred and ninety-six.

17. Add together, 82705 ; — 57926 ;— 6834 ; — 14237 ;— 587 ; — 37645, and 2976.

18. Add together, 617284 :—56392 ;—154027 ;—2465 ;—751270 ; —31286, and 569247.

19. Add together, Forty thousand, seven hundred and ninety-eight ;—twenty-three thousand, six hundred and forty-seven ;—ten thousand, five hundred and seventy-six ;—two hundred and forty-one ;—forty-six thousand, seven hundred and five ;—and eight thousand, one hundred and fifty-seven.

Instead of the former repetition, proceed by adding, as shewn in the following

Example.

572831	Begin as before taught with the lowest figure :
51972	6..10..19..25..27..28..carry 2 ;
317586	2.. 5..13..15..23..30..33..carry 3 ;
4829	3.. 7.. 8..16..21..30..38..carry 3 ;..
643184	3..10..13..17..24..25..27..carry 2 ;
27436	2.. 4.. 8.. 9..14..21..carry 2 ;
———	2.. 8..11..16. The total is 1;617,838
Total 1617838	

EXERCISES.

[20]	[21]	[22]
37185	2170964	21397584
46902	345728	464026
73628	9021863	15843
91037	820715	69531907
57296	43260	8273240
23514	3875419	680139
67295	430736	27863

23. Add together, 4729 ;—683 ;—2405 ;—172 ;—3614 ;—8091, and 726.

24. Add together, Nine hundred and seventy-two thousand, and sixty-four ;—five thousand, four hundred and thirty eight ;— sixty thousand, seven hundred and twelve ;—six hundred and eighty-two thousand, five hundred and forty ;—seventy-two thousand, one hundred and fifty-three ;—and four hundred and seventy-five.

25. Add together, 801729 ;—457064 ;—53297 ;—150362 ;—707291 ;—562873, and 24512.

26. Add together, 90756 ;—3675 ;—43058 ;—294 ;—31279, and 8453.

27. Add together, Six hundred and seventy-five thousand, three hundred and forty-eight ;—nineteen thousand, five hundred and seven ;—two thousand and ninety-five ; three hundred and eighty-one thousand, seven hundred and forty-nine ;—six thousand, one hundred and eight ;—and five hundred and seventy-six.

SIMPLE SUBTRACTION

Teaches to find the difference between two numbers of the same kind by taking the less from the greater.

Example.	
75964	Take 2 from 4, and 2 remain; 3 from 6 and 3
41532	remain; 5 from 9 and 4 remain; 1 from 5 and 4
———	remain; 4 from 7 and 3 remain. The difference
Difference 34432	is 34,432.

When the under figure is greater than the upper, borrow ten, and add one to the next figure in the lower line, as in the following

Example.	
87459	6 from 9, 3;—7 from 5, borrow 10, which added
28176	to 5 make 15—7 from 15, 8;—1 that was bor-
———	rowed added to 1 makes 2—2 from 4, 2;—8 from
Difference 59283	7, borrow 10, 8 from 17, 9;—1 and 2 are 3, 3 from
	8, 5. The difference, or remainder, is 59,283.

Method of Proof. Add the remainder to the less number, and if the sum be equal to the greater, the work is right.

EXERCISES.

	[1]	[2]	[3]	[4]	[5]
From	247	846	7859	2963	53806
Take	123	512	4235	1758	14375
	———	———	———	———	———

	[6]	[7]	[8]	[9]
From	74095	318476	5297631	82765340
Take	32764	307538	189706	45981327
	———	———	———	———

10. From 618963 take 204857.

11. From four hundred and eight thousand, seven hundred and ninety-three—take thirty-nine thousand, eight hundred and fifty-four.

12. From 4070826 take 1265793.

13. From 64072853 take 5197364.

14. From thirty-one millions; four hundred and eighty-six thousand, seven hundred and ninety-two—take two millions; seven hundred and forty-five thousand, three hundred and eighty-six.

SIMPLE MULTIPLICATION

Teaches to increase numbers by repeating them a proposed number of times.

The following Table must be learned *perfectly* before the exercises can be performed.

Twice	3 Times	4 Times	5 Times	6 Times	7 Times
1.... 2	1.... 3	1.... 4	1.... 5	1.... 6	1.... 7
2.... 4	2.... 6	2.... 8	2....10	2....12	2....14
3.... 6	3.... 9	3....12	3....15	3....18	3....21
4.... 8	4....12	4....16	4....20	4....24	4....28
5....10	5....15	5....20	5....25	5....30	5....35
6....12	6....18	6....24	6....30	6....36	6....42
7....14	7....21	7....28	7....35	7....42	7....49
8....16	8....24	8....32	8....40	8....48	8....56
9....18	9....27	9....36	9....45	9....54	9....63
10....20	10....30	10....40	10....50	10....60	10....70
11....22	11....33	11....44	11....55	11....66	11....77
12....24	12....36	12....48	12....60	12....72	12....84

8 Times	9 Times	10 Times	11 Times	12 Times
1.... 8	1.... 9	1.... 10	1.... 11	1.... 12
2....16	2.... 18	2.... 20	2.... 22	2.... 24
3....24	3.... 27	3.... 30	3.... 33	3.... 36
4....32	4.... 36	4.... 40	4.....44	4.... 48
5....40	5.... 45	5.... 50	5.... 55	5.... 60
6....48	6.... 54	6.... 60	6.... 66	6.... 72
7....56	7.... 63	7.... 70	7.... 77	7.... 84
8....64	8.... 72	8.... 80	8.... 88	8.... 96
9....72	9.... 81	9.... 90	9.... 99	9....108
10....80	10.... 90	10....100	10....110	10....120
11....88	11.... 99	11....110	11....121	11....132
12....96	12....108	12....120	12....132	12....144

When the multiplier does not exceed 12.

Example.

Multiplicand 3715642
Multiplier..   2
————
Product.... 7431284

Twice 2 are 4, put down 4 under 2. Twice 4 are 8. Twice 6 are 12, carry 1. Twice 5 and 1 are 11, carry 1. Twice 1 and 1 are 3. Twice 7 are 14, carry 1. Twice 3 and 1 are 7. The product is 7;431,284.

Multiplicand 218574693 | 12 times 3 are 36, carry 3. 12 times 9 and
Multiplier..   12 | 3 are 111, carry 11. 12 times 6 and 11 are
                        | 83, carry 8.   12 times 4 and 8 are 56,
Product....2622896316 | carry 5.   12 times 7 and 5 are 89, carry 8.
                        | 12 times 5 and 8 are 68, carry 6. 12 times
8 and 6 are 102, carry 10.   12 times 1 and 10 are 22, carry 2. 12 times
2 and 2 are 26.   The product is 2,622;896,316.

Method of Proof. The most correct way of proving Multiplication
is by Division; but this cannot be practised until Division is learned.
It may also be proved when the multiplier is large, by changing the
places of the multiplicand and multiplier.

EXERCISES.

1.	Multiply	27043865	by	2.
2.	Multiply	46193754	by	3.
3.	Multiply	537291684	by	4.
4.	Multiply	170839426	by	5.
5.	Multiply	8430716982	by	6.
6.	Multiply	3618092547	by	7.
7.	Multiply	972365014	by	8.
8.	Multiply	280756493	by	9.
9.	Multiply	8761340259	by	10.
10.	Multiply	740381692	by	11.
11.	Multiply	4519728436	by	12.

When the multiplier has two component parts that do
not exceed 12.

Example.

Multiply 31879645 by 72
                 9
         ——————————
         286916805
                 8
         ——————————
Product 2295334440

Here 9 and 8 are the component parts
of 72; because 9 times 8=72. First,
multiply with one figure, as before
taught; then multiply the product with
the other.

The same Example proved.

         31879645
               12
         ——————————
         382555740
                 6
         ——————————
Product 2295334440

Here 12 and 6 are also component parts
of 72, for 12 times 6=72. Hence the
product is 2,295;334,440, the same in
both operations, which proves the work
right.

EXERCISES.

12.	Multiply	4729681	by	24.	Product	113512344.
13.	Multiply	5082763	by	35.	Product	177896705.
14.	Multiply	2368047	by	45.	Product	106562115.
15.	Multiply	824096	by	54.	Product	44501184.
16.	Multiply	107352	by	63.	Product	6763176.
17.	Multiply	635187	by	77.	Product	48909399.
18.	Multiply	1430268	by	84.	Product	120142512.
19.	Multiply	5180376	by	108.	Product	559480608.
20.	Multiply	2173564	by	121.	Product	263001244.
21.	Multiply	8693408	by	132.	Product	1147529856.

22. Multiply eight hundred and forty-nine thousand, three hundred and seventy five, by one hundred and forty-four.

Product 122310000.

When the multiplier consists of several figures, as in the following

Examples.

Multiplicand 8261497
Multiplier.. 3027
: ::

———————

: ::
57830479
: :
16522994
:
24784491
25007551419 Product.

Multiply each line separately, as before taught; carefully observing to place the first figure in the same column as the multiplier; thus 9 is placed under 7;—and 4 under its multiplier 2;—and 1 in the column with its multiplier 3. Then add the several lines together for the Product.

Multiplicand 5308946
Multiplier.. 72358
: : : : :

———————

: : : : :
42471568
: : : :
26544730
: : :
15926838
: :
10617892
:
37162622
384144714668 Product.

Observe, by the dotted lines, how the first figure of every new line is placed beneath its own multiplier. This must be *particularly* attended to.

EXERCISES.

23. Multiply	572638	by	47.	Product	26913986.
24. Multiply	756024	by	85.	Product	64262040.
25. Multiply	219738	by	93.	Product	20435634.
26. Multiply	406523	by	107.	Product	43497961.
27. Multiply	357269	by	235.	Product	83958215.
28. Multiply	819605	by	573.	Product	469633665.
29. Multiply	4076153	by	2085.	Product	8498779005.
30. Multiply	2358746	by	7304.	Product	17228280784.
31. Multiply	5072894	by	40809.	Product	207019731246.
32. Multiply	7098356	by	93584.	Product	664292547904.

33. Multiply one hundred and seventy-four thousand, and fifty-eight, by two hundred and eighty-five.   Product 49606530.

34. Multiply four millions; nine hundred and thirty-seven thousand, two hundred and sixty-eight, by three thousand and fifty-nine.
Product 15103102812.

35. Multiply seventy-four millions; and ninety-six thousand, three hundred and eighty-two, by forty-seven thousand, nine hundred and eight.   Product 3549809468856.

When there are ciphers at the right hand of the multiplicand, or multiplier, or both, as in the following

Examples.

Multiplicand 476283	Neglect the ciphers at the right-hand of both; multiply as before, placing the first figure of every line under its own multiplier; and to the product annex as many ciphers as were neglected.
Multiplier.. 70	
33339810 Product.	

Multiplicand 517960	
Multiplier.. 7400	One cipher in the multiplicand & two in the multiplier neglected—consequently, three ciphers annexed to the product.
207184, 362572	
3832904,000 Product.	

EXERCISES.

36. Multiply   827460   by   90.

37. Multiply   398520   by   5300.

38. Multiply   716300   by   8500.

39. Multiply two hundred and forty-five thousand, by six thousand, nine hundred.   Product 1690500000.

In Multiplication, contractions are made use of, which shorten the work ;—thus

When the multiplier is more than 12, and less than 20, as in the following

Example.

Multiplicand 928346   These multipliers (*viz.* 13, 14, 15,
Multiplier..   18   &c.) may be termed back-figures,
                          because in multiplying, the *back*, or
        16710228 Product. figure behind the one used, is added
                          in. 8 times 6 are 48, carry 4 ;—

8 times 4 and 4 are 36+the back figure 6=42, carry 4; 8 times 3 and 4 are 28+4=32, carry 3; 8 times 8 and 3 are 67+3=70, carry 7; 8 times 2 and 7 are 23+8=31, carry 3; 8 times 9 and 3 are 75+2=77, carry 7; 7 and 9 are 16.   The Product is 16;710,228.

EXERCISES.

40. Multiply   2 3 5 7 6 8   by   13.

41. Multiply   4 7 2 9 3 5   by   15.

42. Multiply   1 3 8 7 0 9   by   17.

43. Multiply   8 0 6 7 2 5   by   19.

44. Multiply   5 7 2 8 0 9 6 by   16.

45. Multiply   9 1 3 7 2 0 8 by   18.

Again, When the multiplier is 21, 31, 41, &c. to 121, as in the following

Example.

Multiplicand 728496   These multipliers may be termed
Multiplier..   91   front-figures, because in multiply-
                          ing, the figure in *front* of the one
        66293136 Product. used, is added in. Bring down the
                          first figure of the multiplicand, *viz.*

6, then 9 times 6 are 54, and front-figure 9=63, carry 6; 9 times 9 and 6 are 87+4=91, carry 9; 9 times 4 and 9 are 45+8=53, carry 5; 9 times 8 and 5 are 77+2=79, carry 7; 9 times 2 and 7 are 25+7=32, carry 3; 9 times 7 and 3 are 66.

The product is 66;293,136.

B 6

EXERCISES.

46. Multiply   4 2 8 6 3 7   by   31.
47. Multiply   5 3 0 7 9 2   by   41.
48. Multiply   7 0 9 5 4 6   by   61.
49. Multiply   2 7 5 3 8 9   by   81.
50. Multiply   1 7 0 6 8 3   by   101.
51. Multiply   8 2 9 6 3 5   by   111.
52. Multiply   3 1 0 7 2 6 9   by   121.

Contractions used.	*Common Method.*
7 8 3 9 6 4	7 8 3 9 6 4
1 9 7 1	1 9 7 1
5 5 6 6 1 4 4 4	7 8 3 9 6 4
1 4 8 9 5 3 1 6	5 4 8 7 7 4 8
	7 0 5 5 6 7 6
1 5 4 5 1 9 3 0 4 4	7 8 3 9 6 4
	1 5 4 5 1 9 3 0 4 4

SIMPLE DIVISION

Teaches to find how often one number is contained in another of the same kind.

When the divisor is not more than 12.

Examples.

Dividend.
Divisor 2) 7431284
Quotient.. 3715642

2 in 7, 3 times and 1 over, place 3 under 7 and carry 1 ; 1 considered as 10 and 4 are 14, 2 in 14, 7; 2 in 3, 1 and 1 over, carry 1 ; 1 considered as 10 and 1 are 11, 2 in 11, 5 and 1 over; 1 considered as 10 and 2 are 12, 2 in 12, 6 ; 2 in 8, 4 ; 2 in 4, 2. The quotient is 3;715,642.

Dividend.
Divisor 12) 465381793

Quotient... 38781816+1

12 in 46, 3 and 10 over; 12 in 105, 8 and 9 over; 12 in 93, 7 and 9 over; 12 in 98, 8 and 2 over; 12 in 21, 1 and 9 over; 12 in 97, 8 and 1 over; 12 in 19, 1 and 7 over; 12 in 73, 6 and 1 over. The quotient is 38;781,816+1 (plus 1).

Method of Proof. Multiply the quotient by the divisor, add the remainder, if any, to the product, and if this product is the same as the Dividend, the work is right.*

---

* The first example in Division is a proof to the first example in Multiplication, see page 7.

EXERCISES.

1. Divide   4 8 6 2 3 5 4 9 0   by   2.
2. Divide   6 0 7 5 8 1 2 4   by   3.
3. Divide   8 2 9 1 0 7 4 6 3   by   4.
4. Divide   9 7 0 3 8 2 6 1 5   by   5.
5. Divide   1 4 8 0 7 2 9 6 3   by   6.
6. Divide   5 2 7 4 3 0 8 2 8   by   7.
7. Divide   3 1 2 1 6 5 4 9 6 2   by   8.
8. Divide   7 0 4 2 8 3 1 4 9 5   by   9.
9. Divide   2 5 1 6 8 4 0 7 6 2   by   10.
10. Divide   6 0 7 2 9 1 5 8 3 8   by   11.

11. Divide thirty-five millions; two hundred and seventy-nine thousand, eight hundred and six, the dividend, by twelve the divisor.

When the divisor has two component parts that do not exceed 12.

Examples.

Divide 2295334440 by 72.

Dividend.

Divisor ⎰ 12)2295334440
72.   ⎱   6) 191277870

Quotient....   31879645

Here 12 and 6 are component parts of 72, because 12×6=72. First divide by one divisor, as before taught, then divide this quotient by the other divisor.*

Divide 8046290534 by 63.

Dividend.

Divisor ⎰ 9)8046290534
63.   ⎱   7) 894032281+5

Quotient..   127718897+2=23 rem.

Here 9 and 7 are the component parts of 63, because 9×7=63. In working this Rule, there are usually two remainders; thus, in the example, 5 is the first remainder, and 2 the last. In order to find the whole remainder, the Rule is—Multiply the *last* remainder with the *first* divisor, and to the product add the first remainder. Hence 2, the last remainder, multiplied with 9, the first divisor=18, and 5, the first remainder, added to this, make 23,—the whole remainder.

EXERCISES.

12. Divide   2351869274   by   18.   Quotient 130659404+ 2.
13. Divide   3197612805   by   32.   Quotient   99925400+ 5.
14. Divide   7514086329   by   48.   Quotient 156543465+ 9.
15. Divide   9068427513   by   66.   Quotient 137400416+57.
16. Divide   3175381602   by   81.   Quotient   39202242
17. Divide   6730815724   by   96.   Quotient   70112663+76.
18. Divide   5463791248   by   121.   Quotient   45155299+69.

19. Divide fifty-seven millions; two hundred and nineteen thousand, and eighty, the dividend, by one hundred and forty-four, the divisor.
Quotient 397354+104.

* This example proves the example in Multiplication, see page 8.

When the divisor has two or more figures, and the foregoing Rules cannot be used, as in the following

Examples.

Divisor.   Dividend.   Quotient.
37) 912763 (24669
     74

     172
     148

     247
     222

     256
     222

     343
     333

Remainder   10

Proof.
24669
   37

172683
74007
   10 Remainder.

912763 Product, the same as the Dividend.

37 in 91 twice, put 2 in the quotient; multiply the divisor by this figure; twice 7 are 14, carry 1; twice 3 and 1 are 7; put 74 under 91, and subtract 4 from 11, 7 remain, carry 1; 1 and 7 are 8, 8 from 9, 1; the remainder is 17.* Bring down the next figure, 2. 37 in 172 are 4; put 4 in the quotient; 4 times 37=148; subtract 148 from 172, the remainder is 24.* Bring down 7. 37 in 247 are 6; put 6 in the quotient; 6 times 37 are 222; subtract 222 from 247, the remainder is 25.* Bring down 6. 37 in 256 are 6. 6 times 37 are 222; subtract 222 from 256, the remainder is 34.* Lastly bring down 3. 37 in 343 are 9; 9 times 37=333; subtract 333 from 343, the remainder is 10.* The quotient is 24669, and 10 remaining.

Divisor.   Dividend.   Quotient.
749) 87594802 (50193
      3745

      1448
       749

      6990
      6741

      2492
      2247

Remainder 245

The first remainder being 14, bring down 4 from the dividend; then 749 in 144 is not sufficient; put 0 in the quotient, and bring down the next figure 8, and proceed as before.

* The remainder must *always be less* than the divisor. Should it be greater, remove the last figure put in the quotient, and place a greater instead.

EXERCISES.

20. Divide   147689   by   31.
21. Divide   520796   by   73.
22. Divide   8371069   by   123.
23. Divide   20458912   by   365.
24. Divide   32704163   by   927.
25. Divide   90258571   by   1435.
           Quotient   62897+1376   remainder.
26. Divide   457609283   by   3047.
           Quotient   150183+1682   remainder.
27. Divide   7213684572   by   8205.
           Quotient   879181+4467   remainder.
28. Divide   30718695274   by   32691.
           Quotient   939668+8686   remainder.
29. Divide 325170913245   by   210472.
           Quotient 1544960+92125   remainder.

30. Divide thirty-one thousand, seven hundred and three millions; four hundred and sixty-five thousand, two hundred and seventy-nine, —by five thousand, three hundred and ninety-eight.

31. Divide five hundred and thirteen thousand, two hundred and nine millions; one hundred and eighty-four thousand, six hundred and fifty-three,—by ninety-seven thousand and sixty-two.

When there are ciphers at the end of the divisor, as in the following

Examples.

	Dividend.	
Divisor 8,00)	2107658,34	Cut them off, and also as many places off the dividend; divide as before, and to the last remainder annex the figures cut off.
	263457+234	

Divisor.   Dividend.   Quotient.
76,000) 594287,463 (7819
      532

      622
      608

      148
      76

      727
      684

     43463 Rem.

Here are three ciphers in the divisor; consequently three figures (463) are cut off the dividend, and are annexed to the remainder.

EXERCISES.

32. Divide   8 3 7 0 4 6 9   by   7 0 0
33. Divide   3 6 9 4 5 7 8 2 6 9   by   3 8 0 0 0
34. Divide   1 2 7 0 4 5 8 9 6 4 7   by   1 3 5 0 0 0

Quotient 94108+9647 rem.

There is a contracted way of performing Long Division (called the Italian method,) which differs only from the common method by omitting the several products.

Example.

Divisor. Dividend. Quotient.
437) 2875961 (6581

    2539

    3546

     501

      64 Remainder.

437 in 2875, 6; 6 times 7 are 42, 42 from 45, 3 remain, carry 4. It is to be observed that 4 is *supposed* to stand before 5, that 42 may be subtracted: and whatever figure is *supposed* to be so placed, is always carried to the next. 6 times 3 and 4 carried are 22; 22 from 27, 5 remain, carry 2: 6 times 4 and 2 are 26; 26 from 28, 2 remain. Bring down 9. 437 in 2539, 5: 5 times 7 are 35; 35 from 39, 4 remain, carry 3; 5 times 3 and 3 are 18; 18 from 23, 5 remain, carry 2: 5 times 4 are 20, and 2 are 22; 22 from 25, 3 remain. Bring down 6; 437 in 3546, 8: 8 times 7 are 56; 56 from 56, 0 and carry 5: 8 times 3 and 5 are 29; 29 from 34, 5 remain, carry 3: 8 times 4 and 3 are 35; 35 from 35, 0. Lastly— Bring down 1; 437 in 501, once; 7 from 11, 4 remain, carry 1; 1 and 3 are 4; 4 from 10, 6 remain, carry 1: 1 and 4 are 5; 5 from 5, 0. The quotient is 6581, and 64 remaining.

Another Example.

Divisor. Dividend. Quotient.
7285) 540729863 (74225

    30779

    16398

    18286

    37163

      738 Remainder.

Proof.

Multiplicand 74225
Multiplier .. 7285

    371125
    593800
    148450
    519575
          738 Remainder.

    540729863 Product.

The foregoing exercises, from No. 20 to the end, may be worked by the contracted method.

TABLES

OF MONEY, WEIGHTS, AND MEASURES.

~~~~~~~~

MONEY TABLE.

2 Farthings.................	1 Halfpenny.
4 Farthings.................	1 Penny.
12 Pence	1 Shilling.
20 Shillings	1 Pound, or Sovereign.

$$\overset{\text{fgs.}}{4} = \overset{\text{D.}}{1}$$

$$48 = 12 = \overset{\text{Sg.}}{1}$$

$$960 = 240 = 20 = \overset{\text{£}}{1}$$

A Half-Crown=2s. 6d.   A Crown=5s.   A Half-Sovereign=10s.

A Guinea=21s.

A Pound or Sovereign, sterling money, is equal to

960 Farthings.	12 Twenty-pences.
480 Halfpence.	10 Two-shillings.
240 Pence.	8 Half-Crowns.
120 Two-pences.	6 Three-shillings & four-pences.
80 Three-pences.	
60 Four-pences.	5 Four-shillings.
40 Six-pences.	4 Crowns.
30 Eight-pences.	3 Six-shillings & eight-pences.
24 Ten-pences.	2 Ten-shillings.
20 Shillings.	

All Accounts are kept in Pounds, Shillings, Pence, and Farthings.

NOTE.—£ denotes Pounds.

S. ...... Shillings.

D...... Pence.

¼ ...... One Farthing.

½ ...... Two Farthings.

¾ ...... Three Farthings.

FARTHINGS TABLE.	PENCE TABLE.		SHILLINGS TABLE.	
d.	*s.*	*d.*	*£*	*s.*
4 fgs.......1	12 pence.. 1	0	20 Sgs. ... 1	0
5 .... 1½	20 1	8	30 1	10
6 .... 1½	24 2	0	40 2	0
7 .... 1¾	30 2	6	50 2	10
8 2	36 3	0	60 3	0
9 .... 2¼	40 3	4	70 3	10
10 .... 2½	48 4	0	80 4	0
11 .... 2¾	50 4	2	90 4	10
12 3	60 5	0	100 5	0
13 .... 3¼	70 5	10	110 5	10
14 .... 3½	72 6	0	120 6	0
15 .... 3¾	80 6	8	130 6	10
16 4	84 7	0	140 7	0
17 .... 4¼	90 7	6	150 7	10
18 .... 4½	96 8	0	160 8	0
19 .... 4¾	100 8	4	170 8	10
20 5	108 9	0	180 9	0
21 .... 5¼	120 10	0	190 9	10
22 .... 5½	130 10	10	200 10	0
23 .... 5¾	132 11	0	210 10	10
24 6	140 11	8	220 11	0
	144 12	0	240 12	0

To find the Pence in any number of Farthings, divide by 4; the quotient will be Pence, & the remainder, Farthings: Thus, 4) 27 fargs.	To find the Shillings in any number of Pence, divide by 12; the quotient will be Shillings; the remainder, Pence. Thus, 12)87 pence.	To find the Pounds in any number of Shillings, divide by 20; the quotient will be Pounds, and the remainder, Shillings. Thus, 2,0)17,5 Sgs.
6.3	7.3	8.15
That is, 27 fgs.=6¾d.	that is, 87 pence=7s. 3d	that is,175 Sgs. = £8.15s.

TROY WEIGHT.

By this weight are weighed Gold, Silver, Jewels, precious Stones, Liquors, and most other things of a fine or costly nature.

THE TABLE.

4 Grains (grs.)	1 Carat............	car.
24 Grains, or 6 Carats....	1 Pennyweight	dwt.
20 Pennyweights	1 Ounce	oz.
12 Ounces.............	1 Pound	lb.

```
Grains.
    4 =      1 Carat.
   24 =      6 =    1 Pennyweight.
  480 =    120 =   20 =   1 Ounce.
 5760 =   1440 =  240 =  12 = 1 Pound.
```

The standard for gold coin is a mixture of 22 carats of fine gold with 2 carats of alloy.

AVOIRDUPOIS WEIGHT.

This weight is used for weighing Bread, Meat, Cheese, Butter, Groceries, Coals, &c., and most of the necessaries of life; also all Metals, except Gold and Silver.

THE TABLE.

16 Drams (dr.).......... 1 Ounce.......... oz.
16 Ounces.............. 1 Pound.......... lb.
 8 lb.................. 1 Stone of Meat.... st.
28 lb.................. 1 Quarter .... qr.
 4 Qrs. or 112 lb........ 1 Hundred-weight .. Cwt.
20 Cwt. .... 1 Ton............ T.

```
Drams.
    16 =       1 Ounce.
   256 =      16 =     1 Pound.
  7168 =     448 =    28 =   1 Quarter.
 28672 =    1792 =   112 =   4 =   1 Cwt.
573440 =   35840 =  2240 =  80 =  20 = 1 Ton.
```

APOTHECARIES WEIGHT

Is chiefly used by Apothecaries and Druggists in compounding medicines.

THE TABLE.

20 Grains (gr.).......... 1 Scruple........ scr.*
 3 Scruples .... 1 Dram.......... dr.
 8 Drams.............. 1 Ounce.......... oz.
12 Ounces.............. 1 Pound .... lb.

* Physicians make use of other characters in writing prescriptions.

Grains.
```
  20 =   1 Scruple.
  60 =   3 =   1 Dram.
 480 =  24 =   8 =   1 Ounce.
5760 = 288 =  96 =  12 = 1 Pound.
```

WOOL WEIGHT

Is used for weighing Wool only.

THE TABLE.

```
 7   Pounds (lb.).......... 1 Clove........ cl.
 2   Cloves, or 14 lb........ 1 Stone........ st.
 2   Stone, or 28 lb......... 1 Tod  ........ td.
 6½  Tods, or 13 stone...... 1 Wey ........ wy.
 2   Weys................. 1 Sack ........ sa.
12   Sacks................. 1 Last ........ la.
```

Pounds.
```
   7 =   1 Clove.
  14 =    2 =   1 Stone.
  28 =    4 =    2 =   1 Tod.
 182 =   26 =   13 =   6½ =   1 Wey.
 364 =   52 =   26 =   13 =   2 = 1 Sack.
4368 =  624 =  312 =  156 =  24 = 12 = 1 Last.
```

CLOTH MEASURE

Is used for measuring Woollens, Linens, Cottons, Silks, &c.

THE TABLE.

```
2¼  Inches (in.)............ 1 Nail.............. nl.
4   Nails ................. 1 Quarter of a yard.... qr.
3   Quarters ............. 1 Flemish Ell........ fl. e.
4   Quarters ............. 1 Yard .............. yd.
5   Quarters ............. 1 English Ell........ e. e.
6   Quarters ............. 1 French Ell ........ fr. e.
```

Inches.
```
2¼ =   1 Nail.
 9 =   4 = 1 Quarter.
36 =  16 =  4 = 1 Yard.
27 =  12 =  3 = 1 Flemish Ell.
45 =  20 =  5 = 1 English Ell.
54 =  24 =  6 = 1 French Ell.
```

LONG MEASURE

Is used for measuring the distance of places from each other.

THE TABLE.

3	Barleycorns (b. c.)........	1 Inch .... in.
12	Inches	1 Foot .... ft.
3	Feet	1 Yard .... yd.
6	Feet	1 Fathom .... fath.
5½	Yards, or 16½ feet	1 Rod, Pole, or Perch...... rd.
40	Poles...................	1 Furlong .... fur.
8	Furlongs, or 1760 Yards ..	1 Mile .... mi.
3	Miles	1 League .... lea.
60	Geographical, or ⎫	
69½	British Miles .. ⎭	1 Degree............... ° or deg.

```
Bar. C.
      3 =      1 Inch.
     36 =     12 =     1 Foot.
    108 =     36 =     3 =     1 Yard.
    594 =    198 =    16½ =    5½ =    1 Pole.
  23760 =   7920 =   660 =   220 =   40 = 1 Furlong.
 190080 =  63360 =  5280 =  1760 =  320 =  8 = 1 Mile.
```

SQUARE, OR LAND MEASURE.

By this measure, Land and all Husbandmen's work are measured; also all kinds of Artificers' work; and all dimensions of Length and Breadth only.

THE TABLE.

144	Square Inches (sq. in.)..........	1 Square Foot...... sq. ft.
9	Square Feet	1 Square Yard..... . sq. yd.
30¼	Square Yards, or ⎫	
272¼	Square Feet.... ⎭	1 Square Pole...... sq. po.
40	Poles.......................	1 Rood............ rd.
4	Roods, or 4840 Square Yards.....	1 Acre .... ac.
10	Chains	1 Acre
640	Acres	1 Square Mile...... sq. mi.

```
        Inches.
           144 =      1 Foot.
          1296 =      9 =     1 Yard.
         39204 =    272¼ =   30¼ =    1 Pole.
       1568160 =  10890 =  1210 =   40 = 1 Rood.
       6272640 =  43560 =  4840 =  160 =  4 = 1 Acre.
```

A square number is produced by multiplying the given number with itself. Thus the square of 4, is 4 × 4, or 16. So also, 12 inches in Long Measure=1 foot; but 12 squared, or 144 inches=1 square foot.

CUBIC, or SOLID MEASURE

Is used in measuring Timber, Stone, &c. and all bodies that have length, breadth, and thickness; also the freight or loading of ships.

THE TABLE.

1728 Cubic Inches (c. in.)............ 1 Cubic Foot...... c. ft.
  27 Cubic Feet.................... 1 Cubic Yard ...... c. yd.
  40 Feet of rough Timber, or }
  50 Feet of hewn Ditto .... } ...... 1 Load or Ton .... l. or t.
  42 Feet........................ 1 Ton of Shipping.

A cubic number is produced by multiplying the given number twice into itself. Thus the cube of 5, is $5 \times 5 \times 5 = 125$. So also 12 inches in Long Measure=1 foot. But 12 cubed, or 1728 inches =1 cubic foot.

WINE MEASURE

Is used for measuring Wines, Spirits, Cider, Oil, Vinegar, Milk, &c.

THE TABLE.

  2 Pints (pt.).................... 1 Quart........... qt.
  4 Quarts....................... 1 Gallon .... gal.
 18 Gallons .... 1 Rundlet......... rund.
 42 Gallons .... 1 Tierce .... tie.
 63 Gallons .... 1 Hogshead .... hhd.
 84 Gallons .... 1 Puncheon .... pun.
126 Gallons .... 1 Pipe or Butt ...... p. or b.
  2 Pipes, or 252 Gallons .... 1 Tun .... t.

    Pints.
       2 =   1 Quart.
       8 =   4 =   1 Gallon.
     336 =   168 = 42 = 1 Tierce.
     504 =   252 = 63 = 1½= 1 Hogshead.
     672 =   336 = 84 = 2 = 1½= 1 Puncheon.
    1008 =   504 = 126 = 3 = 2 = 1½= 1 Pipe.
    2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1 Tun.

Before the Act of Uniformity, which took effect January 1, 1826, there were four different measures of capacity used in England; but these are now equalized by the imperial standard gallon, which contains exactly 10 lbs. avoirdupois of pure water.

BEER MEASURE

Is used for measuring all Malt Liquors.

THE TABLE.

2 Pints (pt.)....................	1 Quart............	qt.
4 Quarts	1 Gallon	gal.
9 Gallons	1 Firkin	fir.
18 Gallons	1 Kilderkin	kil.
36 Gallons	1 Barrel	bar.
54 Gallons	1 Hogshead	hhd.
108 Gallons	1 Butt	butt.
2 Butts, or 216 Gallons	1 Tun	t.

Pints.
```
   2 =   1 Quart.
   8 =    4 =    1 Gallon.
  72 =   36 =    9 =   1 Firkin.
 144 =   72 =   18 =   2 =   1 Kilderkin.
 288 =  144 =   36 =   4 =   2 = 1 Barrel.
 432 =  216 =   54 =   6 =   3 = 1½ = 1 Hogshead.
 864 =  432 =  108 =  12 =   6 = 3 = 2 = 1 Butt.
1728 =  864 =  216 =  24 =  12 = 6 = 4 = 2 = 1 Tun.
```

DRY MEASURE.

By this Measure Corn, Salt, Fruit, Wheat, &c. are measured.

THE TABLE.

2 Pints (pts.)	1 Quart............	qt.
2 Quarts..........................	1 Pottle...........	pot.
2 Pottles, or 4 quarts...............	1 Gallon	gal.
2 Gallons	1 Peck	pk.
4 Pecks	1 Bushel	bush.
4 Bushels	1 Coom...........	coom.
2 Cooms, or 8 Bushels..............	1 Quarter	qr.
5 Quarters	1 Wey, or Load....	wey.
2 Weys...........................	1 Last	l.

Pints.
```
   8 =   1 Gallon.
  16 =    2 =    1 Peck
  64 =    8 =    4 =   1 Bushel.
 256 =   32 =   16 =   4 =   1 Coom.
 512 =   64 =   32 =   8 =   2 =   1 Quarter.
2560 =  320 =  160 =  40 =  10 =   5 = 1 Wey.
5120 =  640 =  320 =  80 =  20 =  10 = 2 = 1 Last.
```

TIME.

THE TABLE.

60	Thirds (‴) .... 1 Second........... 1″
60	Seconds .... 1 Minute. .... 1′
60	Minutes .... 1 Hour .... hr.
24	Hours .... 1 Day .... day.
7	Days....................... 1 Week .... wk.
4	Weeks, or 28 Days........... 1 Month .... mon.
365¼	Days, or 52 Weeks .... 1 Year............. yr.

Seconds.
```
      60 =        1 Minute.
    3600 =       60 =      1 Hour.
   86400 =     1440 =     24 =    1 Day.
  604080 =    10080 =    168 =    7 = 1 Week.
 2419200 =    40320 =    672 =   28 = 4 = 1 Month.
31557600 =   525960 =   8766 =  365¼= 1 Year.
```

> Thirty days hath September,
> April, June, and November ;
> February hath twenty-eight alone,
> And all the rest have thirty-one ;
> Except in Leap Year, then's the time,
> When February's days are twenty-nine.

Leap Year happens every 4th year, and is known by dividing the year by 4, and if there be no remainder, that year is Leap Year.

The Tropical or Solar Year is usually reckoned at 365¼ days ; but its accurate length, as determined from astronomical observations, is 365 days, 5 hours, 48′ 48″.

A Century is 100 years.

MISCELLANEOUS TABLE.

12	Articles, of any kind............... 1 Dozen.
20	Articles .... 1 Score.
12	Dozen .... 1 Gross.
24	Sheets of Paper .... 1 Quire.
20	Quires .... 1 Ream.
2	Reams .... 1 Bundle.

COMPOUND ADDITION

Teaches to add numbers of different denominations, into one whole or total.

Example.

£	s.	d.	
345	12	9¼	Add as taught in Simple Addition, beginning
18	7	11½	with the farthings, thus—
923	19	4¾	2..5..6..8..11..13..14, 14 fgs.=3½d.; put
48	15	7½	down ½, and carry 3 to the pence; 3..6..14..
367	14	1¼	15..22..26..37..46, 46d.=3s. 10d.; put down
43	18	8¾	10, and carry 3 to the shillings; 3..9..17..21
195	6	3½	26..35..42..44; put down 4, and carry 4*—
			4..5..6..7..8..9, 94s.=£4. 14s.; put 1 to the
1943	14	10¼	4, making 14s. and carry 4 to the pounds. The
			pounds are added exactly the same as Simple
			Addition. The total is £1943. 14s. 10¼d.

Method of Proof, the same as in Simple Addition, see page 3.

EXERCISES.

[1]		[2]		[3]		[4]			[5]		
£	s.	£	s.	£	s.	£	s.	d.	£	s.	d.
27	8	370	12	165	18	25	12	6	17	14	8
39	6	65	8	526	12	6	14	2	26	17	11
52	4	147	14	254	14	24	15	5	54	12	5
83	9	306	19	367	16	3	16	4	27	3	10
75	3	43	15	418	19	26	12	3	9	19	7
41	5	971	9	36	11	65	14	7	20	5	9

[6]			[7]			[8]			[9]		
£	s.	d.	£	s.	d.	£	s.	d.		s.	d.
6	12	2½	14	12	8½	405	19	3½		11	10¼
3	18	10¼	9	8	3½	37	15	10½		19	8½
5	6	3	25	19	10¼	4	4	8		4	7¾
8	17	9¾	1	12	5¾	312	18	1¼		18	2
4	14	5½	8	5	11	96	10	11½		15	11¼
2	0	4½	15	13	2½	943	5	0¾		10	10½

* The units figure of the Shillings may always be put down before you add the tens line.

c

	[10]			[11]			[12]	
£	s.	d.	£	s.	d.	£	s.	d.
27	14	9¼	428	3	6	3724	10	11
13	3	11	17	14	8¼	18	13	7¼
4	15	1¾	149	15	11¾	420	5	3½
39	18	0¼	6	18	2½	2705	19	9¼
16	10	4¾	305	7	0	87	7	11
94	7	7½	81	10	10½	356	14	0¾
15	16	8	7	13	3¾	1	15	2¼
3	19	10¾	186	9	1¾	47	8	1¼
87	4	1½	2	15	7	3172	16	9¼

13. A person is indebted to A, £47. 12s. 9¼d.-to B, £300. 7s. 4¼d.-to C, £73. 18s. 11d.-to D, £52. 10s. 7¼d.-to E, £107. 5s. 8¼d.-to F. £41. 10s. 1½d.-and to G, £97. 13s. 3¼d. Required the amount of his debts?   Answer £720. 18s. 10d.

14. E owes F for bread £3. 7s. 2¾d.-for butter £2 18s. 9½d.-for cheese 19s. 10¼d.-for sugar £4. 13s. 7¼d.-for tea £1. 8s. 5d.-and for other articles £18. 17s. 6d.   How much is E indebted altogether?
                                                                       Answer £32. 5s. 5d.

15. A collector of cash gives an account that A paid him £20. 18s. 9d.-B, £8. 7s. 11¾d.-C, £45. 16s. 1½d.-D, £12. 12s. 7¼d.-E, 36. 3s. 4d.-F, £4. 19s. 9½d.-G, £10. 2s. 8d.-and H, £19. 13s. 0¾d. Required the amount of the sums?   Answer £158. 14s. 4d.

16. A gentleman owes to his butcher £53. 17s. 6d.-tallow-chandler, £19. 8s. 3½d.-baker, £117. 10s. 3¾d.-tailor, £42. 5s. 11½d.-brewer, £20. 10s. 7d.-grocer, £31. 17s. 9½d.-cheesemonger, £43. 0s. 9½d.-wine-merchant, £150. 3s. 11d.-and servants' wages, £52. 4s. 7d. Required the amount of his debts?

TROY WEIGHT.

Example.

lb.	oz.	dwts.	grs.
17	3	15	18
3	9	4	16
22	10	17	3
14	5	16	22
9	8	7	14
2	6	13	21
70	8	15	22

1..5..7..10..16..24, carry 2, 2..4..5..7 8..9, making 94 grains; 94÷24 (because 24 grains=1 pennyweight) are 3 and 22 over; carry 3 to the pennyweights; 3..6..13..19.. 26..30..35, carry 3; 3..4..5..6..7, making 75 pennyweights; 75÷20 (because 20 penny-weights=1 ounce) are 3 and 15 over, carry 3 to the ounces; 3..9..17..22..32..41..44; 44÷12 (because 12 ounces=1 pound) are 3 and 8 over, carry 3 to the pounds. The pounds are added as in Simple Addition.
The total is 70 lb. 8 oz. 15 dwts. 22 grs.

EXERCISES.

[17]				[18]				[19]			
lb.	oz.	dwts.		oz.	dwts.	grs.		lb.	oz.	dwts.	grs.
8	5	17		2	15	21		25	8	16	3
2	1	8		11	14	17		4	10	5	18
4	10	13		3	6	12		13	5	19	0
1	7	9		16	13	5		2	9	3	15
3	8	16		2	18	22		28	2	0	22
5	4	11		7	3	9		34	11	17	9

20. A silversmith sold the following quantities of silver, *viz.* 2 lb. 8 oz. 17 dwts. 14 grs.—1 lb. 2 oz. 12 grs.—9 oz. 13 dwts. 15 grs.—14 lb. 10 dwts. 21 grs.—8 oz. 10 dwts. 10 grs.—and 3 lb. 6 oz. 18 dwts. Required the whole quantity?　　　Answer 23 lb. 11 dwts.

AVOIRDUPOIS WEIGHT.

*Example.**

Tons.	cwt.	qrs.	lb.
7	15	2	12
5	7	1	14
2	18	0	25
1	4	3	8
6	19	1	6
3	1	2	26
27	7	0	7

6..12..20..25..29..31, carry 3; 3..5..7..8..9, making 91; 91÷28 (because 28 lb.=1 qr.) are 3 and 7 over, carry 3 to the quarters; 3..5..6..9..10..12; 12÷4 (because 4 qrs.=1 Cwt.) are 3, carry 3 to the Cwt.; 3..4..13..17..25..32..37, carry 3; 3..4..5..6, making 67; 67÷20 (because 20 Cwt.=1 ton) are 3, and 7 over, carry 3 to the tons. Tons, the same as Simple Addition.

The total is 27 tons. 3 cwt. 0 qr. 7 lb.

EXERCISES.

[21]				[22]				[23]			
lb.	oz.	drs.		Cwt.	qrs.	lb.		Cwt.	qrs.	lb.	oz.
125	12	14		2	0	17		2	0	17	13
274	8	10		1	3	19		8	2	25	0
318	15	9		5	2	26		5	1	9	7
42	10	13		4	0	8		0	2	16	14
135	1	8		3	1	12		1	3	14	11
9	14	12		9	3	18		4	0	23	8

* To give an example in each of the following Weights and Measures, would be unnecessary. They are all performed by one principle. Divide the total by as many as make one of the next higher name, put down the remainder, and carry the quotient to the next line.

24. Bought 7 Cwt. 2 qrs. 24 lb. of tea.—12 Cwt. 0 qrs. 19 lb. of coffee.—25 Cwt. 3 qrs. 18 lb. of loaf sugar.—5 Cwt. 1 qr. 22 lb. of spices.—1 Cwt. 2 qrs. 17 lb. of raisins; and 20 Cwt. 3 qrs. 6 lb. of moist sugar. What is the weight of the whole?

Answer 73 Cwt. 2 qrs. 22 lb.

APOTHECARIES' WEIGHT.

Exercises.

[25]			[26]				[27]				
lb.	oz.	dr.	oz.	dr.	scr.	gr.	lb.	oz.	dr.	scr.	gr.
4	9	3	15	3	0	12	1	4	5	1	15
2	10	6	8	7	2	8	5	0	7	0	4
1	3	5	14	2	1	16	4	11	2	2	18
10	4	2	9	6	1	4	6	7	1	1	13
5	11	0	7	0	0	15	2	3	0	2	7
7	6	7	3	5	2	9	7	10	6	0	12

28. A druggist mixed 11 lb. 2 oz. 5 drs.—3 lb. 6 oz. 4 drs.—9 oz. 6 drs.—2 lb. 10 oz. 2 drs.—3 lb. 7 drs.—and 4 lb. 2 oz. 6 drs. together. Required the weight of the composition? Answer 25 lb. 8 oz. 6 dr.

WOOL WEIGHT.

Exercises.

[29]			[30]				[31]		
St.	cl.	lb.	Wy.	td.	st.	cl.	La.	sa.	wy.
18	0	6	5	3	1	0	28	5	1
15	1	4	7	4½	0	1	19	7	1
17	1	2	8	6	1	1	6	11	0
24	0	3	4	2½	0	1	35	3	1
35	1	5	3	2	1	0	17	8	1
14	0	0	5	5	1	1	29	4	0

32. A woolstapler bought 8 sa. 1 wy. 4 tds.—7 sa. 1 wy. 5 tds.—6 sa. 3 tds.—10 sa. 1 wy. 6 tds.—and 12 sa. 1 wy. 2 tds. of wool. What was the whole quantity? Answer 46 sacks. 1 wey. ½ tod.

CLOTH MEASURE.

Exercises.

[33]			[34]			[35]		
Yds	qrs.	nls.	Ells Eng.	qrs.	nls.	Ells. Flem.	qrs.	nls.
25	2	3	26	3	2	24	2	1
52	1	2	17	2	3	17	2	3
17	2	1	25	3	0	15	1	2
26	3	3	16	1	2	16	0	1
14	1	2	28	4	3	34	2	3
27	2	3	14	2	2	9	1	1

36. Bought six parcels of cloth; the 1st contained 37 yds. 1 qr. 2 nls.—the 2nd, 54 yds. 3 qrs. 1 nl.—the 3d, 15 yds. 1 qr. 1 nl.—the 4th, 72 yds. 1 qr. 3 nls.—the 5th, 23 yds. 2 qrs.—and the 6th, 31 yds. 2 qrs. 3 nls. How many yards were purchased?
                                 Answer 235 yds. 0 qrs. 2 nls.

LONG MEASURE.

Exercises.

[37]			[38]			[39]			
Ft.	in.	b. c.	Lea.	mi.	fur.	Mi.	fur.	po.	yds.
74	11	2	19	2	5	7	6	28	4¼
49	10	1	6	2	7	5	2	17	3
37	8	2	35	1	3	0	7	33	2½
15	7	0	18	0	2	3	1	25	1½
8	3	1	14	1	6	4	3	9	5
36	9	2	28	2	4	6	4	14	4½

40. The distance from E to F, is 2 mil. 3 fur. 18 po.—from F to G, 5 mil. 1 fur. 25 po.—from G to H, 7 mil. 7 fur. 12 po.—from H to I, 8 mil. 3 fur. 31 po.—from I to K, 4 mil. 3 fur. 15 po. Required the whole distance from E to K?   Answer 28 mil. 3 fur. 21 po.

SQUARE, or LAND MEASURE.

Exercises.

[41]			[42]			[43]		
Sq. yds.	ft.	in.	Acr.	rd.	po.	Po.	yds.	ft.
175	7	104	28	1	34	5	17½	3
38	3	29	17	3	14	2	25	6
47	5	75	10	1	27	1	13½	7
156	8	131	74	2	19	4	8½	5
81	6	76	63	1	31	3	7	0
235	2	24	55	3	38	6	12¼	4

44. A gentleman had five fields; the first contained 4 acr. 3 rds. 25 po.—the 2d, 6 acr. 0rd. 37 po.—the 3d, 3acr. 1 rd. 20 po.—the 4th, 7 acr. 3 rds. 31 po.—and the 5th, 10 acr. 1 rd. 17 po.   What was the whole number of acres ?   Answer 32 acr. 3 rds. 10 po·

CUBIC, or SOLID MEASURE.

Exercises.

[45]			[46]			[47]		
Yds.	ft.	in.	Yds.	ft.	in.	Yds.	ft.	in.
35	18	35	706	22	91	5	16	939
17 ·	5	21	437	16	8	3	25	1376
40	23	96	593	18	29	4	10	725
59	4	58	406	17	50	1	18	943
37	16	9	28	9	43	2	7	29
5	22	73	591	10	25	1	24	570

WINE MEASURE.

Exercises.

[48]			[49]			[50]			
Hhds.	gal.	qts.	T.	hhds.	gal.	T.	hhds.	gal.	qts.
86	45	3	13	3	15	14	3	27	2
39	57	2	8	1	37	9	2	36	3
47	39	3	14	1	20	17	0	39	2
96	55	0	25	0	12	75	2	16	1
49	48	1	3	2	9	54	1	19	2
58	23	2	12	3	21	97	3	34	3

51. A wine merchant sold 2 hhds. 15 gals. 3 qts. of Sherry.— 5 hhds. 43 gals. 2 qts. of Madeira.—3 hhds. 7 gals. of Bucellas.—7 hhds. 53 gals. 3 qts. of Port—and 4 hhds. 27 gals. 1 qt. of Claret.   How much did he sell in all ?   Answer 23 hhds. 21 gals. 1 qt.

BEER MEASURE.

Exercises.

[52]			[53]			[54]			
Hhds.	gal.	pts.	Kil.	gal.	qts.	Butts.	hhds.	gals.	qts.
71	37	2	7	12	1	1	1	30	3
45	45	5	5	15	3	0	1	18	1
8	29	7	13	6	0	2	0	14	3
16	18	0	4	9	2	1	1	35	2
33	16	6	1	13	1	1	0	49	0
19	3	3	8	7	3	2	1	7	2

55. Received from a brewer the following quantities of beer, *viz.*
5 hhds. 17 gals.—3 hhds. 25 gals. 1 qt.—1 hhd. 8 gals. 2 qts. 1 pt.—
37 gals. 1 qt. 1 pt.—and 2 hhds. 5 gals. 3 qts. I demand the whole
quantity ?   Answer 12 hhds. 40 gals.

DRY MEASURE.

Exercises.

	[56]			[57]			[58]		
Qrs.	bus.	pks.	Bus.	pks.	gal.	Lasts.	qrs.	bus.	pks.
17	7	2	38	2	0	5	7	2	2
89	5	3	17	3	1	7	3	7	3
46	3	1	46	2	1	3	6	5	3
93	4	2	35	3	0	4	9	4	2
5	0	3	92	1	1	6	0	6	0
28	2	0	14	2	0	8	5	3	2

59. Purchased 50 qrs. 3 bus. 2 pks. of Wheat.—7 qrs. 5 bus. 3 pks.
of Oats.—31 qrs. 6 bus. 2 pks. of Rye.—19 qrs. 4 bus. of Barley—and
8 qrs. 7 bus. 3 pks. of Beans. What was the whole quantity ?
                               Answer 118 qrs. 3 bus. 2 pks.

TIME.

Exercises.

	[60]			[61]				[62]		
Wks.	days.	hrs.	Days.	hrs.	′	Days.	hrs.	′	″
8	5	12	37	17	45	5	11	17	55
4	3	18	45	22	37	3	13	42	47
1	4	21	18	9	18	2	9	15	18
0	6	9	19	15	22	1	7	37	13
3	0	13	33	18	51	2	21	9	36
7	2	22	24	20	4	1	18	52	24

COMPOUND SUBTRACTION

Teaches to find the difference between two numbers of different denominations.

Example.

£	s.	d.	
17	12	8	Begin with the lowest name, pence. 3 from 8, 5.
9	15	3	15 from 12 cannot be taken; borrow 20, because
			20 shillings=£1*; 20 added to 12, are 32; 15 from
7	17	5	32, 17. 1, that was borrowed, added to 9, are 10;
			10 from 17, 7. The remainder or difference, is
			£7. 17s. 5d.

Method of Proof is the same as Simple Subtraction.

£	s.	d.	
371	15	2¼	Begin with the farthings. 3 farthings from
236	8	9¾	1 farthing cannot be taken; borrow 4*, because
			4 farthings=1 penny; 4 added to 1 are 5—3 from
135	6	4½	5, 2. 1, that was borrowed, added to 9, are 10;
			10 from 2 cannot be taken—borrow 12*, because

12d.=1 shilling; 12 added to 2 are 14—10 from 14, 4. 1, that was borrowed, added to 8, are 9; 9 from 15, 6. The Pounds are the same as Simple Subtraction. The difference is £135. 6s. 4½d.

EXERCISES.

[1]			[2]			[3]			[4]		
£	s.		£	s.		£	s.	d.	£	s.	d.
27	12		35	6		7	14	8	40	15	4
15	8		12	14		5	9	6	17	10	9

[5]			[6]			[7]			[8]		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
82	12	1	5	18	7½	28	5	9¾	47	16	0¾
33	14	6	3	12	5¼	12	14	6¼	3	11	4¼

[9]			[10]			[11]			[12]		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
21	9	5¼	250	0	8	7	3	5¾	38	0	1
4	17	1½	75	12	6	2	14	7	7	14	3½

* Note that you always borrow as many of the less as make one of the next greater denomination.

13. From £15. 13s. 9½d. take £8. 19s. 7½d.

14. From £172. 16s. 8d. take £94. 18s. 10¾d.

15. Borrowed £25. 14s. 11d., and returned in part £12. 17s. 6½d. What remains due? Answer £12. 17s. 4½d.

16. Lent a person £158. 13s. 9½d. and he has returned £79. 15s. 3¾d. What remains due? Answer £78. 18s. 5¾d.

17. Balance the following Sums.*

£	s.	d.		£	s.	d.
471	18	9½		175	3	0½
26	17	2½		28	17	5½
375	10	0¾		1000	12	3
89	7	11		4	0	9½
300	11	4¾		37	15	11
77	13	8½		2	19	10¾
4	16	5¾		128	8	0½
134	15	7		5	11	4½

Answer £98. 2s. 4½d.

18. Required the balance of the following.

£	s	d.		£	s.	d.
29	14	6		158	3	9¾
452	8	1½		965	19	1¼
3	13	0¾		28	12	6
781	15	11¼		4	7	3¾
33	19	9		158	4	7½
105	12	10½		72	5	10¾
8	7	1¾		206	17	1
53	18	0¼		31	0	9½
348	5	6		9	18	7¼

Answer £182. 5s. 2½d.

19. A merchant is indebted to A £253. 9s. 8½d.-to B, £87. 13s. 10½d. to C, £419. 15s. 11d.—to D, £148. 5s. 5½d.—to E. £83. 0s. 7¾d.— to F, £716. 14s. 7d.—to G, £29. 13s. 3¼d. ; but, upon examining his books, finds that A owes him £59. 4s. 6d.—B, £117. 2s. 11½d.— C, £283. 16s. 8½d.-D, £179. 4s. 3¼d.-E, £41. 5s. 9½d.-F, £280. 10s. and G, £30. 17s. 1½d. Is the balance in his favour? And how much ?

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* The difference between the Totals is the Balance.

TROY WEIGHT.

Example.

lb.	oz.	dwts.	grs.
75	8	16	20
40	10	17	6
34	9	19	14

6 from 20, 14. 17 from 16 cannot be taken—borrow 20 (because 20 dwts.=1 oz.) 20+16 =36; 17 from 36, 19. 1, that was borrowed, added to 10, are 11—11 from 8 cannot be taken; borrow 12, (because 12 oz. = 1 lb.) 12+8=20. 11 from 20, 9. 1 is added to the pounds, and these performed as in Simple Subtraction.

The difference is 34 lb. 9 oz. 19 dwts. 14 grs.

EXERCISES.

	[20]			[21]			[22]		
oz.	dwts.	grs.	lb.	oz.	dwts.	lb.	oz.	dwts.	grs.
17	8	15	291	7	15	503	6	17	21
5	16	21	174	9	12	436	2	17	22

23. A silversmith had 7lb. 8 oz. 9 grs. of silver, and melted down 3 lb. 10 oz. 7 dwts. 15 grs. How much had he remaining?

Answer, 3 lb. 9 oz. 12 dwts. 18 grs.

AVOIRDUPOIS WEIGHT.*

	[24]			[25]			[26]		
lb.	oz.	drs.	Cwt.	qr.	lb.	Cwt.	qr.	lb.	oz.
120	10	4	307	2	10	4	2	13	9
17	10	13	159	3	21	2	3	16	4

27. A grocer bought 18 cwt. 1 qr. 17 lb. of sugar, and made 5 cwt. 2 qrs. 19 lb. into parcels. What quantity remained?

Answer 12 cwt. 2 qrs. 26 lb.

APOTHECARIES' WEIGHT.

	[28]				[29]				[30]			
lb.	oz.	dr.	scr.	oz.	dr.	scr.	gr.	lb.	oz.	dr.	scr.	gr.
4	2	1	0	15	0	0	½	37	3	7	0	14
2	5	2	1	7	6	1	15	19	5	3	1	18

* To give an example in each of the following Weights and Measures would be unnecessary. They are all performed by one principle. When it is required, borrow as many of the less as make one of the next higher name.

31. An apothecary bought 3 lb. 2 oz. 5 drs. 1 scr. of rhubarb, and sold 1 lb. 5 oz. 3 dr.   What quantity had he left?

Answer 1 lb. 9 oz. 2 dr. 1 scr.

WOOL WEIGHT.

EXERCISES.

[32]			[33]				[34]			
La.	sa.	wy.	Sa.	wy.	td.	st.	Wy.	td.	st.	cl.
28	9	1	56	1	6	1	98	4	1	0
16	10	1	37	1	3	0	63	5	1	1

35. Bought 37 sa. 1 wy. 3 tds. of wool, whereof 14 sks. 4½ tds. being damaged; what quantity was fit for use?

Answer 23 scks. 5 tds.

CLOTH MEASURE.

[36]			[37]			[38]			
Yds.	qrs.	nls.	Ells Fr.	qrs.	nls.	Yds.	qrs.	nls.	ins.
174	2	1	171	1	3	32	1	3	1
39	3	2	74	4	2	15	2	1	2

39. From 853 yds.—take 201 yds. 2 qrs. 1 nl. 1 in.

Answer 651 yds. 1 qr. 2 nls. 1¼ in.

LONG MEASURE.

[40]			[41]			[42]			
Lea.	mi.	fur.	Yds.	ft.	in.	Lea.	mi.	fur.	po.
21	2	4	69	0	4	174	2	6	29
17	1	6	27	2	9	58	2	7	33

43. Take 7 mi. 6 fur. 32 po. from 18 mi. 3 fur.

Answer 10 mi. 4 fur. 8 po.

SQUARE, or LAND MEASURE.

Exercises.

[44]			[45]			[46]			
Acr.	rd.	po.	Acr.	rd.	po.	Acr.	rd.	po.	yds.
17	1	14	69	2	13	75	3	19	13
15	2	8	30	3	28	61	3	38	10

47. A person had two pieces of ground: the first measured 19 acr. 3 rd. 17 po.; and the second, 43 acr. 1 rd. 19 po. How much larger was one piece than the other?   Answer 23 acr. 2 rd. 2 po.

WINE MEASURE.

[48]			[49]				[50]		
Tun.	hhds.	gal.	Tun.	hhds.	gal.	qts.	Tier.	gal.	qts.
17	2	23	27	1	20	2	16	18	3
9	1	35	9	0	38	3	7	29	2

51. A cask of wine contained 48 gal. 2 qts.; and 19 gal. 3 qts. 1½ pt. were drawn off. What was the quantity remaining?
Answer 28 gal. 2 qts. ½ pt.

BEER MEASURE.

[52]			[53]			[54]			
Hhds.	gal.	pts.	Bar.	gal.	qts.	Butts.	hhds.	gal.	qts.
42	17	3	37	28	0	7	0	3	1
18	21	6	14	29	2	5	1	18	3

55. A brewer drew 7 hhds. 21 gal. 2 qts. from a large vat, which contained 18 hhds. 30 gal. How much beer remained?
Answer 11 hhds. 8 gal. 2 qts.

DRY MEASURE.

[56]			[57]			[58]			
Qrs.	bus.	gal.	Qrs.	bus.	pks.	Wys.	qrs.	bus.	pks.
15	3	5	87	3	1	121	1	3	2
6	7	6	19	5	2	81	4	2	3

59. From 20 wys.—take 8 wys. 3 qrs. 1 pk.
Answer 11 wys. 1 qr. 7 bus. 3 pks.

TIME.

[60]			[61]				[62]			
Dys.	hrs.	min.	Yrs.	mos.	wks.	dys.	Mos.	wks.	dys.	hrs.
114	17	26	97	7	2	4	8	1	3	9
72	10	37	23	9	3	5	2	2	4	19

63. From 78 months, take 43 mos. 1 wk. 3 dys. 1 hr.

Answer 34 mos. 2 wks. 3 dys. 23 hrs.

COMPOUND MULTIPLICATION

Teaches to increase numbers of different denominations, by repeating them a proposed number of times.

When the Multiplier does not exceed 12.

Examples.

	£	s.	d.
Multiplicand	3	14	8½
Multiplier..			2
Product..	7	9	5

Twice 2 (farthings) 4; 4 farthings=1 penny; carry 1 to the pence. Twice 8 and 1 are 17; 17d.=1s. 5d.; put 5 under the pence, and carry 1 to the shillings. Twice 14 and 1 are 29; 29s.=£1. 9s.; put 9 under the shillings, and carry 1 to the pounds. Twice 3 and 1 are 7.—The Product is £7. 9s. 5d.

	£	s.	d.
Multiplicand	7	15	3¾
Multiplier..			12
Product..	93	3	9

12 times 3 are 36; 36 farthings=9d., carry 9. 12 times 3 and 9 are 45; 45d. =3s. 9d., put 9 under the pence, and carry 3. 12 times 15 and 3 are 183; 183s.=£9. 3s., put 3 under the shillings, and carry 9. 12 times 7 and 9 are 93. The product is £93. 3s. 9d.

Method of Proof is by Compound Division, when learned.

1. 2 lb. of Sugar, at 8½d. per lb.
2. 3 yards of Cloth, at 2s. 3¼d. per yard.
3. 4 lb. of Coffee, at 2s. 9¾d. per lb.

4. 5 gallons of Ale, at 7s. 3d. per gallon.

5. 6 days' Wages, at 6s. 10d. per day.

6. 7 pecks of Corn, at 12s. 5½d. per peck.

7. 8 lb. of Chocolate, at 3s. 9¾d. per lb.

8. 9 lb. of Candles, at 1s. 10½d. per lb.

9. 10 bushels of Oats, at 18s. 7½d. per bushel.

10. 11 ounces of Silver, at 5s. 1½d. per ounce.

11. 12 barrels of Beer, at £1. 13s. 2d. per barrel.

When the Multiplier exceeds 12, and has two component parts.

Example.   84 quarters of Oats, at £1. 11s. 8d. per quarter.

£	s.	d.
1	11	8
		12
19	0	0
		7
133	0	0

12 and 7 are the component parts of 84, because 12 times 7=84. First multiply by 12, and the product again by 7.

EXERCISES.

12. 20 lb. of Tobacco, at 3s. 9½d. per lb.   Answer, £3. 15s. 10d.

13. 33 stone of Beef, at 2s. 7½d. per stone.   Answer, £4. 6s. 7½d.

14. 45 lb. of Tea, at 7s. 10d. per lb.   Answer, £17. 12s. 6d.

15. 54 pints of Wine, at 3s. 2½d. per pint.   Answer, £8. 13s. 3d.

16. 66 yards of Linen, at 5s. 6½d. per yard.   Answer, £18. 4s. 4½d.

17. 81 Cwt. of Cheese, at £2. 17s. 9d. per Cwt.   Ans. £233. 17s. 9d.

18. 90 loads of Hay, at £3. 14s. 2d. per load.   Answer, £333. 15s.

19. 132 bushels of Wheat, at 12s. 5¾d. per bushel.   Ans. £82. 7s. 3d.

When the Multiplier exceeds 12, but has not two component parts.

Example.   53 Cwt. of Sugar, at £3. 6s. 10½d. per Cwt.

£	s.	d.
3	6	10½
		10
33	8	9
		5
167	3	9
10	0	7½
177	4	4½

Multiply by the nearest component parts, and the top line by as many as may be wanted to complete the given number, and add the lower lines together.

Thus—Multiply by 10, and the product again by 5: the top line is multiplied by 3, because 10 times 5 are 50, and 3 added, make the given number.

The same Example proved.

£	s.	d.
3	6	10¼
		9
30	1	10¼
		6
180	11	8
3	6	10¼
177	4	4½

Multiply by 9 and 6, because 9 times 6 =54. But as the required value is one less than 54, subtract the value of 1 Cwt. from the value of 54 Cwt., and the remainder will be the product required.

EXERCISES.

20. 17 yards of Cloth, at 8s. 7d. per yard.   Answer, £7. 5s. 11d.
21. 34 gallons of Oil, at 22s. 9d. per gallon.   Ans. £38. 13s. 6d.
22. 43 acres of Land, at £3. 14s. 5d. per acre.   Ans. £159. 19s. 11d.
23. 67 lb. of Silver, at £3. 8s. 3¼d. per lb.   Ans. £228. 15s. 6¼d.
24. 83 boxes of Indigo, at £1. 19s. 10¼d. per box.
Answer, £165. 7s. 10¾d.
25. 98 Cwt. of Soap, at £2. 3s. 9¾d. per Cwt.   Ans. £214. 13s. 7½d.
26. 127 lb. of Tea, at 8s. 7d. per lb.   Answer, £54. 10s. 1d.
27. 142 yards of Dowlas, at 3s. 11d. per yard.   Ans. £27. 16s. 2d.

When the Multiplier has a fraction, as ½, ⅓, ¾, &c.

Example.   69¾ lb. of Butter, at 1s. 3¼d. per lb.

	s.	d.	
	1	3¼	
		11	
	13	11¾	value of 11 lb.
		6	
	4	3	10¼ value of 66 lb.
	3	9¾	do. 3 lb.
½		7¼	do. ½ lb.
¼		3¾	do. ¼ lb.
	4	8	7¼ do. 69¾ lb.

Multiply as before for the whole numbers, and take parts of the top line for the fraction, and add the values together, 11 times 6 are 66; and the top line by 3; also one-half and one-quarter of the top line; all these values added, give the whole value of 69¾ lb.

The same Example proved.

	s.	d.	
	1	3¼	
		10	
	12	8¼	value of 10 lb.
		7	
	4	8	11½ value of 70 lb.
¼			3¾ do. ¼ lb.
	4	8	7¼* do. 69¾ lb.

10 times 7=70. But the required value is 69¾, being ¼ less than 70. Consequently the value of ¼ lb. (3¾d.) is *subtracted* from the value of 70 lb., and the remainder is the value of 69¾ lb., as required.

---

* The difference of one farthing in the Answers, arises from the omission of the remainders.

28. 7¼ yards of Drugget, at 3s. 8d. per yard.   Answer, £1. 6s. 7d.
29. 24¼ lb. of moist Sugar, at 10½d. per lb.   Answer, £1. 1s. 5¼d.
30. 43¼ yards of rich Silk, at 14s. 7d. yer yd.   Ans. £31. 10s. 8⅜d.
31. 59¾ Cwt. of Lead, at 13s. 9½d. per Cwt.   Ans. £41. 4s. 0¾d.
32. 71½ galls. of Brandy, at 42s. 8d. per gall.   Ans. £152. 10s. 8d.
33. 82¼ ells of Muslin, at 7s. 3¾d. per ell.   Answer, £30. 1s. 5½d.
34. 95¾ firkins of Beer, at 10s. 8d. per firkin.   Ans. £51. 1s. 4d.
35. 113½ Cwt. of Raisins, at £2. 17s. 9½d. per Cwt.
                                        Answer, £327. 19s. 4¼d.
36. 136¼ tons of Iron, at £8. 12s. 5¼d. per ton.
                                        Answer, £1174. 14s. 7¼d.
37. 143¾ lb. of Chocolate, at 4s. 10d. per lb.   Ans. £34. 14s. 9¼d.

WEIGHTS and MEASURES.

Example.

Cwt.	qr.	lbs.	oz.
25	1	13	11
			12
304	1	24	4

12 times 11 are 132.   132÷16, (because 16 oz.=1 lb.) 16 in 132 are 8, and 4 over. 132 oz.=8 lb. 4 oz.   Put 4 under 12, and carry 8 to the pounds.   12 times 13 and 8 are 164.   164÷28, because 28 lbs.=1 quarter.   28 in 164 are 5, and 24 over;   164 lb. = 5 qrs. 24 lb.   Put 24 under the pounds, and carry 5 to the quarters.   12 times 1 and 5 are 17.   17÷4, because 4 qrs. = 1 Cwt.   4 in 17 are 4, and 1 over.   17 qrs. = 4 Cwt. 1 qr. Put 1 under the qrs., and carry 4 to the Cwt.   The Cwt. are performed like Simple Multiplication.
                        The product is 304 Cwt. 1 qr. 24 lb. 4 oz.

Exercises.

38. Multiply 5 lb. 3 oz. 16 dwts. 10 grs. of Silver by 7.
                        Answer, 37 lb. 2 oz. 14 dwts. 22 grs.
39. Multiply 117 yds. 2 qrs. 3 nails of Cloth by 5.
                        Answer, 588 yds. 1 qr. 3 nls.
40. Multiply 58 acr. 1 rd. 25 pls. of Land by 12.
                        Answer, 700 acr. 3 rd. 20 pls.
41. Multiply 5 weys, 1 qr. 7 bus. 2 pks. of Oats by 9.
                        Answer, 48 weys, 2 qr. 3 bus. 2 pks.
42. Multiply 19 tuns, 0 hhd. 19 galls. 2 qts. of Beer by 18.
                        Answer,
43. Multiply 15 weeks, 5 days, 22 hrs. 48 min. by 23.
                        Answer,
44. Multiply 5 Cwt. 2 qrs. 15 lbs. 14 oz. of Lead by 37.
                        Answer,

BILLS OF PARCELS.

[45] A HOSIER'S BILL.

London, August 4, 1843.

MR. JAMES SANDERS,

Bought of CHARLES PERCY.

		s.	d.		£	s.	d.
7 pair of worsted Stockings,	at	3	8	per pair			
4 pair of thread ditto,	at	2	9	....			
5 pair of black silk ditto,	at	13	6	....			
8 pair of black worsted do.,	at	4	0	....			
6 pair of fine cotton ditto..	at	7	3	....			
3 yards of Flannel........	at	1	5	per yard			
9 yards of fleecy Hosiery ..	at	2	7	....			
					£10	7	2

[46] A MERCER'S BILL.

Manchester, September 1, 1843.

MR. JOSEPH KING,

Bought of GEORGE WILLIAMS.

		s.	d.		£	s.	d.
13 yards of Satin	at	8	6	per yard			
17 yards of Velvet	at	15	8	....			
19 yards of Brocade	at	17	3	....			
14 yards of Sarsnet........	at	5	7	....			
20 yards of flowered Satin,	at	11	9	....			
27 yards of Lustring	at	7	3	....			
15 yards of Genoa Velvet ..	at	6	2	....			
					£65	6	0

[47] A LINEN-DRAPER'S BILL.

Liverpool, September 4, 1843.

MR. WILLIAM DAWSON,

Bought of CHARLES LOWE.

		s.	d.		£	s.	d.
27 ells of Dowlas	at	1	3	per ell ..			
43 ells of Diaper	at	1	5¼	....			
40 ells of Holland	at	4	9	....			
38 yards of Irish Linen ..	at	2	7½	....			
25½ yards of Muslin	at	5	8	....			
37 yards of Cambric......	at	11	3	....			
19½ yards of Welch Flannel,	at	1	10	....			
16¼ yards of Cotton	at	1	5	....			
					£50	5	8¾

[48]　A CHEESEMONGER'S BILL.

Islington, September 21, 1843.

Mr. Edward Kendall,

Bought of Samuel Webb.

		s.	d.	£	s.	d.
13	lb. of Stilton Cheese .... at	1	7 per lb.			
5¼	lb. of Cheshire Ditto .... at	0	11			
10¼	lb. of Wiltshire Bacon.... at	0	10¼......			
2	Cheshire Cheeses, wt. 123 lb. at	1	1½......			
17	lb. of fresh Butter ...... at	1	4½......			
A fine Ham, weighing 12¼ lb... at		1	10			
				£10	19	3

[49]　A GROCER'S BILL.

Walthamstow, October 4, 1843.

Mr. James Seddon,

Bought of Isaac Hill.

		s.	d.	£	s.	d.
20	lb. of Muscatel Raisins ... at 1	3	per lb...			
12	cakes of Chocolate .... at 5	7	per cake			
23	lb. of best loaf Sugar ..... at 1	1½	per lb .			
24¼	lb. of moist Sugar........ at 0	8¼	....			
39	lb. of Rice.............. at 0	4½	....			
10	lb. of Hyson Tea .... at 7	3	....			
14¼	lb. of Black Tea .... at 6	4	....			
15¾	lb. of Coffee .... at 2	8	....			
				£17	13	9½

[50]　A WINE-MERCHANT'S BILL.

London, October 18, 1843.

Stephen Jones, Esq.

Bought of Edward Pope.

		s.	d.	£	s.	d.
5	gallons of Brandy...... at 34	6	per gall.			
13½	dozen of Port .... at 40	6	per doz.			
6	ditto of Claret .... at 5	9	per bottle			
17½	ditto of Sherry .... at 36	0	per doz.			
4¼	gallons of Gin .... at 20	3	per gall.			
9	dozen of Bucellas...... at 3	7	per bottle			
10¼	ditto of Lisbon .... at 42	6	per doz.			
3	gallons of Rum........ at 34	7	per gall.			
				£139	6	3¾

COMPOUND DIVISION

Teaches to find how often one number is contained in another of different denominations.

When the Divisor does not exceed 12.

Examples.

£	s.	d.	
Divisor 2) 7	9	5	Dividend.
Quotient.. 3	14	8½	

2 in 7, 3 and 1 over. £1 over= 20s. and 9 (in the dividend) are 29; 2 in 29 are 14 and 1 over. 1s. over=12d. and 5 are 17; 2 in 17, 8 and 1 over. 1d. over=4 farthings, 2 in 4, 2.

The Quotient is £3. 14s. 8½d.

Method of Proof is by Compound Multiplication.*

£	s.	d.	
Divisor 12) 93	3	9	Dividend.
Quotient.. 7	15	3¾	

12 in 93, 7 and 9 over; £9 over =180s. and 3 are 183. 12 in 183 are 15 and 3 over. 3s. over=36d. and 9 are 45. 12 in 45, 3 and 9 over. 9d. over=36 farthings, 12 in 36, 3.

The Quotient is £7. 15s. 3¾d.

EXERCISES.

1. Divide £8. 14s. 9d. by 2.	Answer, £4. 7s. 4½d.
2. Divide £13. 8s. 7½d. by 3.	Answer, £4. 9s. 6½d.
3. Divide £35. 15s. 4½d. by 4.	Answer, £8. 18s. 10d. +2.
4. Divide £42. 7s. 0¾d. by 5.	Answer, £8. 9s. 4¾d.+4.
5. Divide £19. 15s. 10d. by 6.	Answer, £3. 5s. 11½d.+4.
6. Divide £85. 3s. 3½d. by 7.	Answer, £12. 3s. 3¾d.+5.
7. Divide £17. 2s. 4½d. by 8.	Answer, £2. 2s. 9½d.+2.
8. Divide £43. 18s. 10d. by 9.	Answer, £4. 17s. 7½d.+1.
9. Divide £15. 3s. 7½d. by 10.	Answer, £1. 10s. 4½d.+3.
10. Divide £58. 0s. 10¾d. by 11.	Answer, £5. 5s. 6½d.+8.
11. Divide £123. 17s. 9d. by 12.	Answer, £10. 6s. 5¾d.

* These two Examples prove the two in Compound Multiplication—(see page 37.)

When the Divisor exceeds 12, and has two component parts.

Example.   What is Soap per lb. when 56 lb. cost £2. 4s. 4d. ?

```
      £   s.  d.
   8) 2   4   4
   ─────────────
    7)  5   6½
   ─────────────
             9½
```

Take the component parts, and first divide by one part, and then the quotient by the other. Here 8 and 7 are the component parts of 56. 8 times 7=56.

EXERCISES.

12. If 28 pieces of cloth cost £57. 18s. 9d. ; what is that per piece ?
Answer, £2. 1s. 4½d.+12.

13. Suppose the wages of 42 labourers for a week come to £75. 15s. 6d. ; what is each man paid?   Answer, £1. 16s. 1d.

14. Let £125. 10s. be equally divided between 64 persons; what is the share of each ?   Answer, £1. 19s. 2½d.+32.

15. What is the value of a hogshead of Beer, when 120 are sold for £148. 5s. ?   Answer, £1. 4s. 8½d.

If the Divisor exceed 12, and has not two component parts.

Example.   Divide £68. 5s. 7¾d. by 19.

```
      £   s.  d.  £.  s.   d.
  19) 68   5   7¾ (3  11   10½
      57
     ────
      11
       20 shillings=£1.

  19) 225 (11 shillings.
      209
     ─────
       16                    This is performed after the manner
        12 pence=1 shilling.      of Long Division.

  19) 199 (10 pence.
      190
     ─────
        9
         4 farthings=1 penny.

  19) 39 (½d.
      38
     ────
        1 Remainder.          The Quotient is £3. 11s. 10½d.+1.
```

Another Example. If a person's income be £452. 18s. 6d. what can he spend per day ?

```
        £    s.   d. £   s.   d.
365) 452   18   6 (1   4   9¾
        ___
        87
        20 shillings=£1.
        ____
365) 1758 (4 shillings.
        ____
        298
        12 pence=1 shilling.
        ____
365) 3582 (9 pence.
        ___
        297
        4 farthings=1 penny.
        ____
365) 1188 (¾d.
        ____
        93 Remainder.
        ====
```

Divide by 365, as 365 days= 1 year. Contracted Division is here used, as taught in Simple Division—(see page 16).

Answer, £1. 4s. 9¾d.+93.

EXERCISES.

16. Divide £32. 15s. 7¼d. by 23.   Answer, £1. 8s. 6d.+5.
17. Divide £198. 3s. 6d. by 57.   Answer, £3. 9s. 6¼d.+39.
18. Divide £576. 18s. 9½d. by 123. Answer, £4. 13s. 9½d.+116.
19. Divide £1408. 17s. 8d. by 247. Answer, £5. 14s. 0¾d.+203.
20. If 61 oxen cost £492. 13s. 8½d., what is the price of each ?

Answer, £8. 1s. 6¼d.+45.

21. If 112 lb. of cheese cost £3. 13s. 6d., what is that per lb. ?

Answer, 7¾d.+56.

22. A quantity of coffee, weighing 253 lbs., cost £33. 4s. 1¼d., what was the cost per lb. ?   Answer, 2s. 7½d.

23. Suppose 279 lb. of tobacco cost £23. 18s. 4d., what is that per lb. ?   Answer, 1s. 8¼d.+82.

24. Paid £1308. 19s. 7½d. for 453 cwt. of cheese, required the charge per cwt. ?

WEIGHTS AND MEASURES.

Example.   Divide 124 yards 3 qrs. 2 nls. by 37.

```
        yds. qrs. nls.  yds. qr. nls.
  37 )  124   3   2  ( 3   1   2
        111
        ───
         13
          4 qrs. = 1 yard.
        ───

  37 )    55 ( 1 qr.                Quotient, 3 yds. 1 qr. 2 nls.
          37
        ───
          18
           4 nails = 1 qr.

  87 )    74 ( 2 nls.
          74
        ───
           0
        ═══
```

EXERCISES.

24. Divide 18 lb. 5 oz. 13 dwts. 14 grs. by 5.
       Answer, 3 lb. 8 oz. 6 dwts. 17 grs. + 1.

25. Divide 24 Cwt. 2 qrs. 17 lb. 12 oz. by 9.
       Answer, 2 Cwt. 2 qrs. 26 lb. 13 oz. + 7.

26. Divide 384 miles, 1 fur. 25 poles, by 17.
       Answer, 22 mls. 4 fur. 32 pls. + 1.

27. Divide 175 yds. 3 qrs. 1 nl. by 36.
       Answer, 4 yds. 3 qrs. 2 nls. + 5.

28. Divide 47 acres, 2 roods, 14 poles, by 51.
       Answer, 3 roods, 29 poles + 15.

29. Divide 118 lasts, 1 qr. 5 bus. 2 pks. by 83.
       Answer, 1 last, 4 qrs. 1 bus. 3 pks. + 49.

30. Divide 42 yrs. 5 mos. 3 wks. 5 dys. by 12.
       Answer, 3 yrs. 6 mos. 1 wk. 6 dys. + 10.

31. Divide 136 Cwt. 3 qrs. 10 lb. 11 oz. 9 drs. by 23.
       Answer, 5 Cwt. 3 qrs. 22 lb. 6 oz. 1 dr. + 2.

32. Divide 37 hhds. 23 galls. 3 qts. by 15.

33. Divide 407 yds. 2 qr. 1 nl. by 39.

34. Divide 37 lb. 1 oz. 5 dwts. 22 grs. by 47.

EXERCISES IN THE PRECEDING RULES.

1. Required the difference between one hundred millions, three hundred and seven thousand, and ninety-five—and two millions, and forty-six thousand, three hundred and eight.

2. Divide eight thousand and seventy-three millions, one hundred and sixty-five thousand, two hundred and four, by twenty-seven thousand and ninety-five.

3. A ship was freighted with 14 tons, 17 cwt. 2 qrs. 15 lb. of sugar—22 tons, 13 cwt. 0 qrs. 27 lb. of tobacco—3 tons, 8 cwt. 3 qrs. 22 lb. of indigo—3 tons, 16 cwt. 1 qr. 9 lb. of hemp—16 tons, 2 cwt. 3 qrs. 25 lb. of flax—and 7 tons, 17 cwt. 3 qrs. 11 lb. of iron. What was the ship's burden ?

4. Express in figures, seventy-five millions, four hundred and three thousand, and fifty-two—three thousand and ninety-one millions, two hundred and sixty-five thousand, and forty—and eight billions, two hundred and three thousand and seven millions, one hundred and eighty-three thousand, two hundred.

5. Required the total value of 18 lb. of sugar, at 8¼d.—47 lb. of tea, at 6s. 3d.—9¼ lb. of coffee, at 3s. 5d.—16½ lb. of loaf sugar, at 10¾d.—22¼ lb. of currants, at 7¼d.—and 7¾ lb. of raisins, at 8½d.

6. A gentleman's income is 800 guineas; at the end of the year he finds he has expended for rent £125—for taxes, £30. 17s. 9d.—butcher's bills, £153. 19s. 3¾d.—baker's bills, £139. 15s. 11½d.—grocer's bills, £37. 4s. 5d.—cheesemonger's bills, £23. 0s. 9½d.—wine-merchant's bills, £47. 8s. 3d.—tailor's bills, £25. 7s.—wages to his servants, £61. 18s.—subscriptions to charitable institutions, 30 guineas—and various petty expences, £15. 2s. 9½d.—What has he remaining ?

7. £757. 8s. 4¼d. are to be divided between 23 men, required the share of each ?

8. Lent 100 guineas, and received in return £48. 13s. 3¼d. What remains due ?

9. Add together two hundred and eighty-seven thousand, and fifty-seven—five thousand, four hundred, and eighty-six—thirty-three thousand, five hundred, and ninety-two—one hundred and forty-seven thousand, four hundred, and three—fifty thousand, one hundred, and eighty-six—eight thousand, and five—and four hundred and seventy-three thousand, one hundred, and ninety-six.

10. £5. 13s. 4d. were distributed between 17 men. What was the share of each ?

11. A man's wages are 45 shillings per week, and his expences average 41s. 9¼d. How much can he save yearly?

12. According to astronomical calculation, the Earth is ninety-five millions; and Sirius, the brightest of the stars, thirty-two billions of miles distant from the Sun. Express their distances in figures?

13. Borrowed £600.—and have returned £149. 17s. 2¼d.—£78. 16s. 9¼d.—50 guineas—£48. 9s. 10½d.—£86. 3s. 7d.—£104. 14s. 8½d.—£35. 2s. 1¼d. What remains due?

14. Multiply fifty-seven millions, three hundred and ninety-two thousand, and seventy-eight, by forty-seven thousand, nine hundred, and six.

15. A silversmith bought 47 lb. 8 oz. of silver: he made 5 lb. 10 oz. 3 dwts. into tea-pots—3 lb. 4 oz. 15 dwts. 17 grs. into table-spoons—4 lb. 11 oz. 7 dwts. 19 grs. into tea-spoons—6 lb. 3 oz. 5 dwts. 9 grs. into dish-covers—and 5 lb. 8 oz. 13 dwts. 22 grs. into mugs. What quantity had he remaining?

16. Add together 547218, 13097, 28216, 917380, 28163, 47182, and 573; and from the total subtract nine hundred and seventy thousand, two hundred, and eighty-five.

17. Bought tea at £26. 16s. 8d. per Cwt. What is the price per lb.?

18. Required the whole value of 12 yards of silk at 15s. 2d.—25¼ yds. of cloth, at 1s. 8½d.—15 yds. of satin, at 3s. 7d.—41¼ yds. of linen, at 2s. 7½d.—and 48½ yds. of ribbon, at 9¼d.

19. Multiply two millions, eight hundred and seven thousand, six hundred, and thirty-nine, by fifty thousand, four hundred, and seventy-two; and divide the product by twelve thousand, six hundred, and eighteen.

20. What is the weight of 43 hogsheads of sugar, each 3 cwt. 2 qr. 17 lb.?

21. Divide five billions, seven hundred and eight thousand, and four millions, two hundred and eighty-six thousand, and seventy-one, by seven hundred and forty-nine thousand, and sixty-three.

22. Multiply eighty-seven millions, two hundred and ninety-six thousand, four hundred and five, by five hundred and eighty-four thousand, and ninety-seven.

REDUCTION

Teaches to change numbers of one name into others of another name, without altering their value.

RULE I.—Great names are brought into small, by *multiplying* by as many of the small as make one of the great.

RULE 2.—Small names are brought into great, by *dividing* by as many of the small as make one of the great.

Example. Reduce £3246. 18s. 7d. into shillings and pence.

£	s.	d.
3246	18	7
20		

64938 shillings.*
12

779263 pence.*

To reduce pounds into shillings and pence, is to change great names (pounds) into small (shillings and pence).

By Rule 1. Multiply the pounds by 20, because 20 shillings=1 pound, taking in 18 shillings. Next, multiply the shillings by 12, because 12 pence=1 shilling, taking in 7 pence.

Answer, 64938 shillings, and 779263 pence.

Example.

lb.	oz.	dwts.	grs.
17	5	13	17
12			

209 ozs.*
20

4193 dwts.*
24

16789
8386

100649 grains.*

In 17 lb. 5 oz. 13 dwts. 17 grs. of silver, how many grains?

To reduce pounds into grains, is to bring great names (lb.) into small (grs.)

Multiply by 12, because 12 ozs. = 1 lb.; next by 20, because 20 dwts.=1 oz.; lastly, by 24, because 24 grs.=1 dwt.

Answer, 100649 grains.

EXERCISES.

1. Reduce £62. 13s. 9d. into shillings and pence.
   Answer, 1253 sgs. 15045 pence.
2. In £2735 how many crowns and shillings?
   Answer, 10940 crowns. 54700 sgs.
3. Reduce 427 guineas into shillings, pence, and farthings.
   Answer, 8967 sgs. 107604 pence. 430416 farthings

---

* Always affix the names to the several products.

4. In £5027 how many crowns and half-crowns?
> Answer, 20108 crowns. 40216 half-crowns.

5. Reduce £265. 17s. 10¼d. into farthings.
> Answer, 255257 farthings.

6. In £372. 18s. 6d. how many sixpences?*
> Answer, 14917 sixpences.

Example. In 12214 farthings how many pence, shillings, and pounds?

Farthings.
4) 12214
————
12) 3053¼
————
20) 25,4   5
————
£12 14   5½

> To reduce farthings into pence, shillings, and pounds, is to change small names (farthings) into great (pence, shillings, and pounds).
>
> By Rule 2. Divide by 4, because 4 farthings =1 penny. Divide the pence by 12, because 12 pence=1 shilling. Divide the shillings by 20, because 20 shillings=1 pound.

Answer, 3053¼ pence, 254 shillings 5¼d., and £12. 14s. 5½d.

Example.   Reduce 12843499 drams into tons.

Drams.
16 { 4) 12843499
     ————
     4) 3210874+3
     ————
16 { 8) 802718+2=11 drs.
     ————
     2) 100339+6
     ————
28 { 7) 50169+1=14 ozs.
     ————
     4) 7167
     ————
     4) 1791+3=21 lb.
     ————
     20) 44. 7+3 qrs.
     ————

> To reduce drams into tons, is to change small names (drams) into great (tons).
>
> Divide by 16, because 16 drs.=1 oz. Again, by 16, because 16 oz.=1 lb. Next, by 28, because 28 lb. = 1 qr. Next, by 4, because 4 qrs.=1 cwt. Lastly, by 20, because 20 cwt.=1 ton.

Tons. 22. 7 cwt. 3 qrs. 21 lb. 14oz. 11 drs.

> Answer, 22 tons. 7 cwt. 3 qrs. 21 lb. 14 ozs. 11 drs.

EXERCISES.

7. In 71854 pence how many shillings and pounds?
> Answer, 5987 sgs.+10d.   £299. 7s. 10d.

8. Reduce 207563 farthings into pence, shillings, and pounds.
> Answer, 51890¾ pence, 4324 sgs. 2¾d. £216. 4s. 2¾d.

9. Reduce 29560 sixpences into half-crowns and pounds.
> Answer, 5912 half-crowns, 739 pounds.

---

* Multiply by the number of sixpences in a pound.

10. In 25643 twopences how many shillings and pounds?
Answer, 4273 sgs. 10d. £213. 13s. 10d.

11. Reduce 38716 half-crowns into crowns and pounds.
Answer, 19358 crowns. £4839. 10s.

12. In 283526 farthings how many pounds?
Answer, £295. 6s. 9½d.

Examples, combining Rule 1st and Rule 2d.

Reduce 5178 guineas into shillings and pounds.

Guineas.
5178
21*
_____
2,0)10873,8 shillings.
_____
£5436,18

To reduce guineas into shillings, is to change great names into small. By Rule 1st, multiply by 21, because 21 shillings = 1 guinea. Again, to reduce shillings into pounds, is to change small names into great. By Rule 2d, divide by 20, because 20 shillings = 1 pound.

Answer, 108738 sgs. and £5436. 18s.

In £8205. 14s. how many pieces, each £3. 12s. ?

£ s.   £ s.
3 12   8205 14
20   20
____   ____
72   ) 164114 (2279
          201
          571
          674

Remainder 26 sgs. = £1. 6s.

The pounds are brought into shillings, and then divided by 72 shillings, the value of each piece.

N. B. Two numbers cannot be divided, unless in the same name. 26 shillings over, as the Remainder is always of the same name as the Dividend.

Answer, 2279 pieces + £1. 6s.

EXERCISES.

13. Reduce £8701. 15s. into shillings and guineas.
Answer, 174035 shillings, 8287 guineas, 8s.

14. In 7254 moidores, each 27 shillings, how many shillings and pounds?   Answer, 195858 shillings. £9792. 18s.

15. Reduce 1874 pieces, each 5s. 6d., into sixpences and pounds.
Answer, 20614 sixpences. £515. 7s.

16. In 21587 guineas how many shillings and moidores?
Answer, 453327 shillings. 16789 moid. 24 sgs.

17. In £6285. 15s. how many sixpences, and pieces each 4s. 6d.?
Answer, 251430 sixpences. 27936 pieces, 3s.

18. How many pounds are in 25070 dollars, each 4s. 2d.?
Answer, £5222. 18s. 4d.

_____

* Multiplied in one line, as taught in the Contractions of Simple Multiplication, page 11.

EXERCISES IN MONEY, WEIGHTS, AND MEASURES.

19. Reduce 8739 half-guineas into pence and farthings.
                    Answer, 1101114 pence, 4404456 farthings.
20. In £213. 14s. how many ducats, each 4s. 9d. ?
                    Answer, 899 ducats, 3s. 9d.
21. Reduce 59 lb. 9 oz. 13 dwts. of silver into grains.
                    Answer, 344472 grains.
22. In 37 cwt. 2 qrs. 17 lb. how many pounds?
                    Answer, 4217 pounds.
23. In 26173 crowns how many shillings and guineas?
                    Answer, 130865 shillings. 6231 guineas, 14s.
24. Reduce 20571500 drams into tons.
                    Answer, 35 tons. 17 Cwt. 1 qr. 25 lb. 6 oz. 12 drs.
25. Reduce 37 miles. 3 fur. 28 poles. 2 yds. 2 ft. 9 in. into inches.
                    Answer, 2373729.
26. In 17 pieces of Cloth, each containing 24 Flemish ells, how
many yards?   Answer, 306 yards.
27. In 26 Cwt. 2 qr. 17 lb. of Tobacco, how many parcels, each
15 lb. ?   Answer, 199 parcels.
28. How many spoons can be made, each to weigh 3½ ounces,
from 215 lb. 8 oz. of Silver?   Answer, 739 spoons, 1½ oz.
29. In 75 hogsheads of Wine how many gallons and pints?
                    Answer, 4725 gallons, 37800 pints.
30. From the 7th of May, 1843, to the 19th February, 1844, both
days inclusive, how many days?   Answer, 289 days.
31. Reduce 41729 farthings into pence and dollars, each 4s. 5d.
                    Answer, 10432¼ pence. 196 dollars, 3s. 8¼d.
32. Reduce 97 yards, 3 qrs. into ells English.
                    Answer. 78 ells, 1 qr.
33. How many seconds are in a solar year, or 365 days, 5 hours.
48 min. 55 sec. ?   Answer, 31556935 seconds.
34. Reduce £4079 into shillings, and pieces each £3. 12s.
                    Answer, 81580 sgs. 1133 pieces. 4s.
35. In 13 hogsheads of Sugar, each 18 Cwt. 2 qrs. 17 lb., how
many pounds?   Answer, 27157 lb.
36. If a person step at an average 2½ feet, how many steps will he
take in walking 15 miles?   Answer, 31680 steps.
37. In 17 hogsheads of Tobacco, each weighing 15 Cwt. 2 qrs.
14 lb. how many parcels, each 17½ lb. ?
38. How many hours have elapsed since the birth of Christ to the
present day inclusive, allowing the year to consist of 365¼ days?
39. Bought 23 pockets of Hops, each weighing 1 Cwt. 2 qr. 17 lb.,
how many pounds?
40. Required the number of days and hours from the 17th of
October, 1831, to the 8th of April, 1844, both days exclusive.

THE RULE OF THREE,*

OR

SIMPLE PROPORTION,

Teaches to find a number that shall have the same proportion to one of the three given numbers, as there is between the other two.

RULE.

1st. Ascertain, from the question, whether the fourth term, or answer, is to be money, weight, or measure, &c., and put the number of the same kind for the third term.

2nd. Consider whether the fourth term, or answer, is to be greater or less than the third : if greater, put the greater of the two remaining terms in the second place ; but if less, the less ; and, in both cases, the other for the first.

3d. Bring the first and second terms, when necessary, into the same name, and the third into the lowest denomination mentioned.

4th. Multiply the second and third terms together, and divide the product by the first, and the quotient will be the fourth term, or answer, in the same denomination the third term was left in.

Note. It is often required to bring the quotient into a higher name, that the answer may be exhibited in its proper form.

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* This Rule was formerly divided into the Rule of Three Direct, and the Rule of Three Inverse. Now, most Arithmeticians give a general Rule, applicable to all questions.

Example. If 5 lb. of Sugar cost 4s. 9½d., what cost 29 Cwt. 3 qrs. 16 lb. ?

1st term.	2d term.	3d term.
5 lb. :	29 C. 3 qrs. 16 lb. : :	4s. 9½d.
	4	12

119 qrs.	57 pence.
28	4

968	230 fgs.
238	

3348 lb. ⎱ 2d and 3d terms
　 230 ⎰　multiplied.

100440
6696

1st term. 5)770040 product.

4)154008 quotient in farthings.

12)38502

2,0)320,8 . 6

£160.8 . 6

Answer, £160. 8s. 6d.

By Rule 1st. The 4th term must be in money, the question requiring the *Cost* of a certain quantity of Sugar. 4s. 9½d. is put in the third term.

2d. The 4th term must be *greater*, because the required value of 29 Cwt. 3 qrs. 16 lb. will be more than the given value of 5 lb. (4s. 9½d.); consequently, the greater is put in the second term, and the other in the first.

3d. The 1st & 2d terms are brought into the same name (lb.); and the 3d is brought into the lowest denomination mentioned (farthings).

4th. The 2d & 3d terms are multiplied together, and the product divided by the 1st. The quotient (154008) is in the same denomination as the 3d term was left in (farth.). This quotient is reduced into pence, shillings, and pounds, to exhibit the Answer clearer.

Example. If 54 men finish a piece of work in 24 days, how many men would be required to perform the same work in 18 days ?

18 days : : 24 days : : 54 men.
　　　　　24

　　　　216
　　　　108

　　18)1296(72

　　　　36

　　　　　0

Answer, 72 men.

By Rule 1st. The 4th term must be *men*, the question being, " how many men?"

2d. The 4th term must be *greater*, because it would require more men to perform a piece of work in 18 days, than it would in 24 days.

3d. Bring, &c. Here the 1st and 2d terms are in the same name (days).

4th. Multiply, &c. The quotient is 72 men, the same denomination the 3d term was left in.

Note. The operation can sometimes be shortened by dividing the 1st and 2d terms by a common divisor.

18 days : 24 days : : 54 men.
6—   6—   4
  3   4   —————
  =   =   3) 216
                   —————
                      72

Here the 1st and 2d terms are divided by 6, which divides both 18 and 24, without a remainder in either.

Answer, 72 men.

Example. How many yards of Linen can I purchase for 76 guineas, at 4s. 10d. per ell English ?

4s. 10d. : 76 guis. : : 1 ell Eng.
12   21
—   —————
58d.   1596 shillings.
  =   12
            —————
58) 19152d. (330 ells Eng.
    —————
    175
    —————
     12
      5 qrs. =1 ell. Eng.
    —————
58) 60 (1 qr.
      2
     ═══
          E. E.   qr.
          330   1
            5
          —————
       4) 1651 qrs.
          —————
          412 yds. 3 qrs.

Answer, 412 yards, 3 qrs.

By Rule 1st. The 4th term must be in measure, the question being, "how many *yards?*"

1 ell English is put in the 3d term.

2d. The 4th term must be *greater*, because 76 guineas will purchase more linen than 4s. 10d. will.

3d. The 1st and 2d terms are brought into pence. The 3d does not require to be reduced.

4th. The 3d term (1 e. E.) need not be multiplied with the 2d; but the 2d is taken as the product, and divided by the 1st.

As the question requires the answer to be exhibited in yards, the quotient is (by the rules of Reduction) changed from ells English to yards.

Note. The Rules for performing Interest, Commission, Discount, Fellowship, Barter, Loss and Gain, Exchanges, &c. are all based upon the Rule of Proportion; so that if a thorough knowledge of stating the question be acquired, the others follow of course. From its utility and universal application, this Rule was anciently called, The Golden Rule.

EXERCISES.

1. If 1 pound of Tobacco is sold for 2s. 7d., what is the value of 1 Cwt.?   Answer, £14. 9s. 4d.

2. Purchased 1 yard of Cloth for 3s. 9½d.; how much must be given for 38 yards, 3 qrs.?   Answer, £7. 6s. 11d.

3. How many yards of Cloth, 3 quarters wide, are equal in measure to 30 yards, of 5 quarters wide?   Answer, 50.

4. Paid 2s. 3d. for a pint of Wine; what quantity can be had for 12 guineas, at the same rate?   Answer, 14 gallons.

5. If 1 pound of Rice cost 4½d., what cost 18 Cwt. 2 qrs. 17 lb?
                    Answer, £39. 3s. 4½d.

6. Required the Wages due for 27 days' service at £18. 18s. per year, reckoning 365 days?   Answer, £1. 7s. 11½d.

7. If the carriage of goods at 1¼d. per pound came to £37. 16s. 9½d.; I demand the weight?   Answer, 64 Cwt. 3 qrs. 13 lb.

8. If 14 men can dig a trench in 18 days, in how many days will 34 men perform the same?   Answer, 7 days, 9 hours.

9. Required the value of 37 Cwt. 1 qr. 16 lb. of Coffee, if 1 lb. cost 2s. 3½d.?   Answer, £479. 17s. 6d.

10. What is the value of a piece of Cloth, containing 57½ yards, at 3s. 7½d. per yard?   Answer, £10. 8s. 5¼d.

11. A gentleman spends one day with another £1. 7s. 10½d., and lays by annually £170.; what is his yearly income?
                    Answer, £678. 14s. 4½d.

12. If the carriage of 18 Cwt. 1 qr. 17 lb. cost £27. 18s. 7½d.; required the charge per lb?   Answer, 3½d.

13. How much Matting, 5 feet broad, will cover a floor 23 feet long and 28 broad?   Answer, 128⅘ feet.

14. At 13s. 6d. per week, how long can I be boarded for £35?
                    Answer, 51 weeks, 5 days.

15. Bought an ounce of Silver for 5s. 7½d.; what is the value of a Tankard weighing 117 oz. 13 dwts.?   Answer, £33. 1s. 9½d.

16. Lent a friend £200 for 9 months; how long ought he to lend me £150, to return the kindness?   Answer, 12 months.

17. How many Cwt. of Soap can I buy for 7½ guineas, if 1 lb. is sold for 7¾d.?   Answer, 2 Cwt. 19 lb.

18. If £100 yield £3. 10s. yearly interest, what will be the interest of £1450. 15s. ? Answer, £50. 15s. 6¼d.

19. Purchased 4 yards of Linen for 17s. 4d.; what cost 235 ells English, 1 qr. ? Answer, £63. 14s.

20. If 23 horses can be kept in a meadow for 18 days, how long would the same meadow maintain 14 horses ? Answer, 29 days.

21. Gave 4 guineas for 1 ounce of Gold; what is the value of a grain ? Answer, 2d.

22. What is the yearly rent of 257 acres of Land, at 17s. 6d. per acre ? Answer, £224. 17s. 6d.

23. If a servant's wages be 14 guineas per annum, what must be paid for 9 weeks, 3 days' service? Answer, £2. 13s. 1¾d.

24. How many pounds of Tea, at 7s. 3d. per lb., can I have for 15 Cwt. 7 lb. of Coffee, at 3s. 9d. per lb.? Answer, 872 lb.

25. If 54 men can build a house in 96 days, how many would be required to build it in 36 days? Answer, 144.

26. What is the value of 175 tons, 15 Cwt. of Coals, at £2. 8s. 7d. per ton ? Answer, £426. 18s. 6¼d.

27. Sold a parcel of old Silver for 25 guineas, at 5s. 7¼d. per ounce: I demand the weight of it? Answer, 7 lbs. 9oz. 6 dwts. 16 grs.

28. At 5s. 8¼d. per bushel, how many quarters of Corn can I buy for £47. 15s. 6d. ? Answer, 20 qrs. 7 bus. 1 pk.

29. If the carriage of 20 Cwt. for 15 miles come to £1. 4s. 7d., how far can I have 8 Cwt. 3 qrs. 16 lb. carried for the same money ?

Answer, 33 miles, 5 fur.

30. How many pieces of money, each 4s. 8¼d., can be had in exchange for 275 pieces, each 3s. 9d. ? Answer, 219.

31. Suppose a gentleman's income is £500, and he spends 23s. 6d. one day with another, how much will he save yearly?

Answer, £71. 2s. 6d.

32. If 1 ell English cost 5s. 10d., what cost 427½ yards?

Answer, £99. 13s. 10d.

33. If a person travel 12 hours a day, and finish his journey in 36 days; how long would the same journey take him if he travelled only 9 hours a day at the same rate?

Answer, 48 days.

34. Bought 18 pieces of Cloth, each containing 27 yards, at the rate of 4s. 11d. per yard. Required the value of the whole?

Answer, £119. 9s. 6d.

35. If a certain number of men can raise a wall in 10 days, when they work 12 hours a-day, how many days would the same wall require in building, if the men worked 8 hours?   Answer, 15.

36. If £5. 3s. 9d. is paid for 23 yds. 3 qrs., what cost 137 ells English, 1 qr.?   Answer, £37. 9s. 2d.

37. How much do I spend yearly, if I spend 17s. 6½d. per day?

Answer, £320. 2s. 8½d.

38. How many yards of Stuff, 3¾ yards wide, are equal in length to a piece 17 yards long, and 2¼ wide?   Answer, 10 yds. 3 nls.

39. What cost 42 qrs. 3 bus. of Wheat, at £1. 9s. 6d. per quarter?

Answer, £62. 10s. 0¾d.

40. If a wall 7 ft. 8 in. high, cast a shadow which is 5 ft. 4 in. in length, required the height of a steeple casting a shadow 138 ft. 7 in. in length?   Answer, 199 ft. 2 in.

41. Bought 3 tuns of Oil for £195. 17s. 6d.; but 75 gallons being damaged, I am willing to sell the remainder at a certain price per gallon, so as neither to lose nor gain by the sale. Required the selling price per gallon?   Answer, 5s. 9d.

42. How much Chocolate, at 3s. 8d. per lb., can I have for 12 Cwt. 1 qr. 4 lb. of Rice, at 5¼d. per lb.?

Answer, 1 Cwt. 1 qr. 24 lb.

43. Paid 17s. 9d. for 7 yards of Linen; what must be given for 5 pieces, each containing 27½ yards?

Answer, £17. 8s. 7¾d.

44. How many pieces of Coin of 7s. 3d. in value, are equal to 1172 pieces of 4s. 8½d. in value?   ·   Answer, 761 pieces.+11d.

45. A merchant failing in business, owes £3726. 15s. 9d.; but his effects amount only to £848. 7s. 1d.; what must his creditors receive per pound?   Answer, 4s. 6½d.

46. Purchased 17 pieces of old Lead, each weighing 1 Cwt. 2 qrs. 13 lb., at 9s. 8½d. per Cwt: required the value?

Answer, £13. 6s. 8½d.

47. If 1 ounce of Coffee cost 2¼d., what quantity can be had for £13. 5s.?   Answer, 88 lb. 5 oz.

48. Suppose a servant is paid £8. 15s. 3d. for 7 months 3 weeks service, how much is that per year (365 days)?

Answer, £14. 14s. 9¼d.

49. If Tea is sold at 7s. 9d. per pound, what is the price of 3 chests, weighing as follows:—No. 1, 3 Cwt. 1 qr. 20 lb.; No. 2, 2 Cwt. 2 qr· 14 lb.; No. 3, 4 Cwt. 0 qr. 7 lb.?   Answer, £439. 0s. 9d.

50. A person failing in business, pays 7s. 3d. in the pound; what must be received for a debt of £158. 13s. 9d.?

Answer, £57. 10s. 5¾d.

51. How many Cwt. of Tobacco, at 2s. 10d. per lb., can be had for 32 guineas?   Answer, 2 Cwt. 13 lb.

52. Bought 13 Cwt. 1 qr. 15 lb. of Coffee for £180. 17s. 3½d.; but 173 lb. being damaged, how must the remainder be sold per lb. so as not to lose by the accident?   Answer, 2s. 8½d.

53. If 1 Cwt. 3 qr. 18 lb. of Sugar cost £9s. 4s. 7½d., what quantity may be had for £473. 13s. 9¼d.?   Answer, 98 Cwt. 5 lb.

54. If 5 oz. 14 dwts. of Gold are sold for £25. 13s., what is the value of 10 lb. 5 oz. 13 dwts. 21 grs.?   Answer, £565. 12s. 5½d.

55. Suppose 10 lb. 5 oz. 13 dwts. 21 grs. cost £565. 12s. 5½d., what cost 5 oz. 14 dwts?   Answer, £25. 13s.

56. Bought 5 oz. 14 dwts. of Gold for £25. 13s.; what quantity can I have for £565. 12s. 5½d.?   Answer, 10 lb. 5 oz. 13 dwts. 21 grs.

57. How many Cwt. of Sugar can be purchased for 35 guineas, at 9½d. per lb.?

58. If 3½ yards of Cloth were sold for 12s. 1½d., what would be the cost of 7 pieces, each 25¾ yards?

59. If a meadow maintain 21 cows for 15 days, how long would the same meadow keep 35 cows?

60. A merchant bought 4 hogsheads of Sugar, weighing as follows: No. 1, 3 Cwt. 2 qrs. 12 lb.; No. 2, 2 Cwt. 0 qr. 25 lb.; No. 3, 4 Cwt. 3 qrs. 15 lb.; No. 4, 3 Cwt. 1 qr. 19 lb.; at the rate of £2. 11s. 7d· per Cwt. Required the value of the whole?

COMPOUND PROPORTION

Is a Rule by which the student may resolve such questions as require two or more statings in Simple Proportion.*

Example. If 7 men in 12 days reap 126 acres, in how many days will 16 men reap 72 acres?

The Answer must be *days;* consequently, days must be noticed in both statings.

1st. If 7 men in 12 days reap —— acres, in how many days will 16 men reap —— acres?

Here, by Rule 2d in Simple Proportion, the less number (7) is put in the second term, because 16 men could reap a certain number of acres in less time than 7 men could: thus,

$$16 \text{ men} \quad : \quad 7 \text{ men} \quad : : \quad 12 \text{ days.}$$

2d. If —— men in 12 days reap 126 acres, in how many days will —— men reap 72 acres?

Here again the less number (72) is put in the second term, because 72 acres could be reaped by a certain number of men in less time than 126 acres could:—thus,

$$126 \text{ acres} \quad : \quad 72 \text{ acres} \quad : : \quad 12 \text{ days.}$$

Now, combining the statings,

$$\left.\begin{array}{lll} 16 \text{ men} & : & 7 \text{ men} \quad : : \\ 126 \text{ acres} & : & 72 \text{ acres} \quad : : \end{array}\right\} 12 \text{ days.}$$

By Rule 4th, Simple Proportion, multiply the second and third terms together, and divide by the first.

---

* The method of reasoning in the Examples is the same as pursued in Simple Proportion. It may be proper to remark, each Example is divided into two parts, *only* to render the reasoning plainer. In performing the Exercises, the learner may dispense with this in placing the terms.

By actual Operation.

$$\left.\begin{matrix}72\\7\end{matrix}\right\} \text{2d terms.}$$

$$\begin{matrix}504\\12 \quad \text{3d term.}\end{matrix}$$

$$6048$$

1st term
16 $\left\{\begin{matrix}4) \ 6048 \\ 4) \ 1512\end{matrix}\right.$

also,
1st term 126) 378 (3 days.
　　　　　　　378

　　　　　　　　0

Answer, 3 days.

By Cancelling.

$$\frac{72 \times \cancel{7} \times 12}{16 \times \underset{18}{\cancel{126}}} = \frac{\overset{4}{\cancel{72}} \times 12}{16 \times \cancel{18}}$$

$$= \frac{\overset{\cancel{4}}{} \times 12}{\underset{4}{\cancel{16}}} = \frac{12}{4} = 3 \text{ days.}$$

Example. If a family of 8 persons expend £200 in 9 months, how much will serve a family of 18 people 12 months?

The Answer must be money; consequently, money must be noticed in both statings.

1st. If a family of 8 persons expend £200 in —— months, how much will serve a family of 18 people —— months?

Here, the greater (18) is put for the 2d term, because 18 persons will spend more money in a certain number of months than 8 persons will;—thus,

　　　　　　　8 persons : 18 persons : : £200
or, dividing by 2, 4 persons : 9 persons : : £200

2d. If a family of —— persons expend £200 in 9 months, how much will serve a family of —— people 12 months?

Again, the greater (12) is put for the second term, because a certain number of persons will spend more in 12 than in 9 months: thus,

　　　　　　　9 months : 12 months : : £200
dividing by 3, 3 months : 4 months : : £200

Combining these statings—

$$\left.\begin{matrix}\text{1st, 4 persons} & : & \text{9 persons} & : : \\ \text{2d, 3 months} & : & \text{4 months} & : :\end{matrix}\right\} \text{£200}$$

Multiply the 2d and 3d terms together, and divide by the 1st.

$$\frac{9 \times \cancel{4} \times 200}{\cancel{4} \times 3} = \frac{\overset{3}{\cancel{9}} \times 200}{\cancel{3}} = \text{£600}$$

Answer, £600.

Example. If the carriage of 11 Cwt. 2 qrs. for 150 miles cost
£6. 14s. 8d., how much must be paid for the carriage of 15 Cwt.
1 qr. 22 lb. for 64 miles at the same rate?

The Answer must be money, consequently, money must be noticed
in both statings.

1st. If the carriage of 11 Cwt. 2 qrs. for —— miles cost £6. 14s. 8d.,
how much must be paid for the carriage of 15 Cwt. 1 qr. 22 lb. for
—— miles at the same rate?

Here, the greater (15 Cwt. 1 qr. 22 lb.) is put for the 2d term,
because the carriage of 15 Cwt. 1 qr. 22 lb. for a certain distance
would cost more than the carriage of 11 Cwt. 2 qrs.;—thus,

    11 Cwt. 2 qrs. :   15 Cwt. 1 qr. 22 lb. : :   £6. 14s. 8d.
  or,   1288 lb. :   1730 lb.   : :   1616 pence.

2d. If the carriage of —— Cwt. &c. for 150 miles cost £6. 14s. 8d.,
how much must be paid for the carriage of —— Cwt. &c. for 64
miles at the same rate?

Here the less (64) is put for the 2d term, because the carriage of
a certain weight 64 miles, would cost less than the carriage of the
same weight 150 miles:—thus,

    150 miles   :   64 miles   : :   1616d. (£6. 14s. 8d.)

Combining the statings,

    1st, 1288 lb. : 1730 lb.   : : }
    2d,   150 miles : 64 miles : : } 1616 pence.

Multiply the 2d and 3d terms together, and divide by the 1st.

    1288 1st term.   1730 2d term.
     150 1st term.   64 2d term.
    ————   ————
   193200   6920
                                 10380
                               110720
                                1616 3d term.
                               ————
                             1771520
                             1771520

                 1932,00) 1789235,20 (926 pence.

   12) 926 pence   5043
   ————   ————
  20) 77 . 2   11795
  ————   ————
Answer, £3.17.2+20320   20320 Remainder.

EXERCISES.

1. If 27 shillings be the wages of 4 men for 7 days, what will be the wages of 14 men for 10 days?   Answer, £6. 15s.

2. If 2 acres of land will maintain 3 horses for 4 days, how long will 5 acres of the same pasture maintain 6 horses?   Answer, 5 days.

3. Suppose £100 in 12 months gain £6 interest, how much will £75 gain in 9 months?   Answer, £3. 7s. 6d.

4. If 12 men consume 32 pounds of bread in 18 days, how much will 24 men consume in 36 days?   Answer, 128 lb.

5. How many men can complete a trench of 135 yards long in 8 days, when 16 men can dig 54 yards of the same trench in 6 days?
Answer, 30 men.

6. If the carriage of 126 lbs. for 100 miles cost 6 shillings, how many pounds can I have carried 750 miles for a guinea?
Answer, 58¼ lbs.

7. How many bushels of Oats will be sufficient for 20 horses 24 days, if 14 horses eat 56 bushels in 16 days?   Answer, 120 bushels.

8. If a footman travel 130 miles in 3 days, when the days are 12 hours long; in how many days, of 10 hours each, will he travel 360 miles?   Answer, 9$\frac{69}{75}$ days.

9. If 2 men in 3 days can earn 15 shillings, how much can 7 men earn in 4 days?   Answer, £3. 10s.

10. What principal will gain £20 interest in 19 months, if £100 gain £4 interest in 12 months?   Answer, £315. 15s. 9¼d.¦$\frac{7}{9}$

11. If a barrel of beer be sufficient to last a family of 8 persons 12 days, how many barrels will be drank by 16 persons in a year?
Answer, 60$\frac{5}{6}$ barrels.

12. If a regiment of soldiers, consisting of 939 men, consume 351 quarters of Wheat in 168 days, how many soldiers will consume 1404 quarters in 56 days?   Answer, 11268 soldiers.

13. If 8 men in 14 days can mow 112 acres of grass, how many men must be employed to mow 200 acres in 10 days?

14. If 9 men cast 30 yards of ditching in 12 days, working 8 hours a-day, how many men will cast 45 yards in 18 days, working 6 hours per day?

PRACTICE

Is a Rule generally used in business, as a compendious method of performing the Rule of Three, when the first term is unity.

It is performed by taking aliquot parts of the price, and dividing those parts into the given quantity.

TABLE OF ALIQUOT PARTS.*

Of a Penny.	Of a Pound.	Of a Cwt.
$\frac{1}{2}$d. .. is .. $\frac{1}{8}$	10s. .. is .. $\frac{1}{2}$	2 qrs. .. is .. $\frac{1}{2}$
$\frac{1}{4}$d. .. — .. $\frac{1}{4}$	6s. 8d. — .. $\frac{1}{3}$	1 qr. .. — .. $\frac{1}{4}$
	5s. .. — .. $\frac{1}{4}$	16 lb. .. — .. $\frac{1}{7}$
Of a Shilling.	4s. .. — .. $\frac{1}{5}$	14 lb. .. — .. $\frac{1}{8}$
6d. .. is .. $\frac{1}{2}$	3s. 4d. — .. $\frac{1}{6}$	8 lb. .. — .. $\frac{1}{14}$
4d. .. — .. $\frac{1}{3}$	2s. 6d. — .. $\frac{1}{8}$	7 lb. .. — .. $\frac{1}{16}$
3d. .. — .. $\frac{1}{4}$	2s. .. — .. $\frac{1}{10}$	
2d. .. — .. $\frac{1}{6}$	1s. 8d. — .. $\frac{1}{12}$	Of a Quarter.
1$\frac{1}{2}$d... — .. $\frac{1}{8}$	1s. .. — .. $\frac{1}{20}$	14 lb. .. is .. $\frac{1}{2}$
1d. .. — .. $\frac{1}{12}$	8d. — .. $\frac{1}{30}$	7 lb. .. — .. $\frac{1}{4}$
	6d. — .. $\frac{1}{40}$	4 lb. .. — .. $\frac{1}{7}$
	4d. — .. $\frac{1}{60}$	3$\frac{1}{2}$ lb. .. — .. $\frac{1}{8}$
	3d. — .. $\frac{1}{80}$	2 lb. .. — .. $\frac{1}{14}$
	2d. — ..$\frac{1}{120}$	

1st. When the price is less than a penny.

Example.

```
d.            1705 at ¼
¼..¼          ———
         12) 426¼
         ———————
         2,0) 3,5 . 6
         ———————
              £1.15.6¼
```

Take aliquot parts of a penny,—divide the quotient by 12, and by 20. This is the same as reducing farthings into pounds.

---

* *Aliquot* means an exact part of a certain number—thus 1½d. is an aliquot part of a shilling (one-eighth); but 1¾d. is *not* an aliquot part.

Example.

2739 at ¾d.

d.

½··½   1369 . ½
¼··¼   684 . ¾

12) 2054 . ¼

2,0) 17,1. 2

£8.11.2¼

A halfpenny is the ½ of a penny, and a farthing is the ½ of a halfpenny. Add— and divide by 12 and 20 as before.

Proof.

2739 at ¾d.
      3

4) 8217

12) 2054 . ¼

2,0) 17,1 . 2

£8.11 . 2¼

Multiply by the number of farthings, and reduce the product into pounds.

EXERCISES.

1. 1237 lb. at ¼d.	Answer, £1. 5s. 9¼d.
2. 2349 lb. at ½d.	Answer, £4. 17s. 10½d.
3. 2795 lb. at ¾d.	Answer, £8. 14s. 8¼d.
4. 1521 lb. at ¾d. .	Answer, £4. 15s. 0¾d.
5. 3203 lb. at ½d.	Answer, £6. 13s. 5½d.
6. 2987 lb. at ¼d.	Answer, £3. 2s. 2¾d.

2d. When the price is less than a shilling.

Example.

3795 at 4½d.

d.

4··⅓   1265
½··⅛   158 . 1½

2,0) 142,3 . 1½

£71.3 . 1½

Take parts of a shilling;—thus, 4d. is ⅓, and ½d. is ⅛ of 4d. Add—and divide by 20 for the pounds.

Proof, another way.

3795 at 4½d.

d.
3..¼   948 . 9
1½.½   474 . 4½

2,0) 142,3 . 1½

£71.3 . 1½

Here, 3d. is the ¼ of a shilling, and 1½d. is the ½ of 3d.

Example.

529 at 10¾d.

d.
6... ½   264 . 6
3... ½   132 . 3
1½.. ½   66 . 1½
¼.. ⅙   11 . 0½

2,0) 47,3 . 10¾

£23.13 10¾

Here, 6d. is the ½ of a shilling,—3d. is the ½ of 6d.,—1½d. is the ½ of 3d.,—and ¼d. is the ⅙ of 1½d.

NOTE.—In working Pratice, always take the parts to make the full money, before dividing each line.

Proof, another way.

529 at 10¾d.

d.
6..½   264 . 6
4..⅓   176 . 4
¾..⅛   33 . 0¾

2,0)47,3 .10¾

£23.13.10¾

Here, 6d. is the ½ of a shilling, 4d. is the ⅓ of a shilling, and ¾ is the ⅛ of 6d.

This method is frequently shorter, but requires care in dividing the proper line.

EXERCISES.

7. 357 oz. at 1d.   Answer, £1. 9s. 9d.
8. 493 oz. at 1¼d.   Answer, £2. 11s. 4¼d.
9. 715 oz. at 1½d.   Answer, £4. 9s. 4½d.
10. 307 oz. at 1¾d.   Answer, £2. 4s. 9¼d.
11. 948 oz. at 2d.   Answer, £7. 18s. 0d.
12. 742 oz. at 2¼d.   Answer, £6. 19s. 1½d.
13. 159 oz. at 3d.   Answer, £1. 19s. 9d.
14. 640 oz. at 3½d.   Answer, £9. 6s. 8d.
15. 541 oz. at 4d.   Answer, £9. 0s. 4d.

16.	833 oz. at 4¾d.	Answer, £16. 9s. 8¾d.
17.	1051 oz. at 5d.	Answer, £21. 17s. 11d.
18.	817 oz. at 5½d.	Answer, £17. 17s. 5½d.
19.	2143 oz. at 6d.	Answer, £53. 11s. 6d.
20.	574 oz. at 6¼d.	Answer, £15. 10s. 11d.
21.	908 oz. at 7d.	Answer, £26. 9s. 8d.
22.	635 oz. at 7¾d.	Answer, £20. 10s. 1¼d.
23.	1329 oz. at 8d.	Answer, £44. 6s. 0d.
24.	617 oz. at 8¾d.	Answer, £22. 9s. 10¾d.
25.	574 oz. at 9½d.	Answer, £22. 14s. 5d.
26.	803 oz. at 10d.	Answer, £33. 9s. 2d.
27.	476 oz. at 10¼d.	Answer, £20. 6s. 7d.
28.	359 oz. at 11d.	Answer, £16. 9s. 1d.
29.	645 oz. at 11¼d.	Answer, £30. 4s. 8¼d.
30.	1174 oz. at 11¾d.	Answer, £57. 9s. 6½d.

3d. When the price is a certain number of shillings under 20.

Example.

479 at 5s.
5s..¼ ————-
£119.15

If it is an aliquot part of a Pound, divide, as in the Example.

Example.

573 at 17s.
17
————
2,0) 974,1
————
£487.1

If it is not an aliquot part, multiply by the number of shillings, and divide the product by 20, for Pounds.

EXERCISES.

31.	723 dwts. at 2s.	Answer, £72. 6s.
32.	587 dwts. at 3s.	Answer, £88. 1s.
33.	401 dwts. at 4s.	Answer, £80. 4s.
34.	928 dwts. at 6s.	Answer, £278. 8s.
35.	589 dwts. at 7s.	Answer, £206. 3s.
36.	615 dwts. at 9s.	Answer, £276. 15s.
37.	1301 dwts. at 10s.	Answer, £650. 10s.

38.   850 dwts. at 11s.   Answer, £467. 10s.

39.   783 dwts. at 12s.   Answer, £469. 16s.

40. 2057 dwts. at 15s.   Answer, £1542. 15s.

41. 1416 dwts. at 18s.   Answer, £1274. 8s.

42.   375 dwts. at 19s.   Answer, £356.   5s.

4th. When the price is shillings, pence, and farthings.

Example.
1057 at 4s. 8¾d.

4s...⅕	211 . 8	
6d...⅛	26 . 8.6	
2d...⅓	8 . 16 . 2	
¾....⅜	3 . 6.0¾	

£249 . 18 . 8¾

Take parts of a pound; thus,
4s. is the ⅕, 6d. is the ⅛th of 4s.,
2d. is the ⅓ of 6d., and ¾d. is the
⅜ of 6d.

Example.
705 at 17s. 10¾d.

10s.....	½	352 . 10
5s......	½	176 . 5
2s. 6d...	½	88 . 2 . 6
3d...	₁⁄₁₀	8 . 16 . 3
1½d..	½	4 . 8 . 1½
¼d..	⅙	0 . 14 . 8¼

£630 . 16 . 6¾

Proof, by taking different parts.
705 at 17s. 10¾d.

				s. d.
10s...	½	352 . 10	=705 at	10 . 0
6s. 8d.	⅓	235 . 0	=705 at	6 . 8
1s....	₁⁄₂₀	35 . 5	= do. at	1 . 0
2d. ..	⅙	5 . 17 . 6	= do. at	0 . 2
½d. ..	¼	1 . 9 . 4½	= do. at	0 . 0½
¼d. ..	½	0 . 14 . 8¼	= do. at	0 . 0¼

£630 . 16 . 6¾ = do. at 17.10¾

Exercises.

43.   813 yds. at 1s. 8d.   Answer, £67. 15s. 0d.

44.   275 yds. at 1s. 10½d.   Answer, £25. 15s. 7½d.

45.   408 yds. at 2s. 3d.   Answer, £45. 18s. 0d.

46. 1371 yds. at 2s. 9¼d.   Answer, £189. 18s. 9¾d.

47.   937 yds. at 3s. 4d.   Answer, £156. 3s. 4d.

48.   185 yds. at 3s. 7¾d.   Answer, £33. 14s. 5¾d.

49.   433 yds. at 4s. 2d.   Answer, £90. 4s. 2d.

50. 2079 yds. at 4s. 8½d.   Answer, £487. 5s. 3¾d.

51.   873 yds. at 5s. 6d.   Answer, £240. 1s. 6d.

52.   665 yds. at 6s. 3½d.   Answer, £209. 3s. 11½d.

53.   721 yds. at 7s. 4¼d.   Answer, £266. 12s. 4¾d.

54.	249 yds. at	9s.	5d.	Answer, £117. 4s. 9d.
55.	1406 yds. at	10s.	8½d.	Answer, £752. 15s. 11d.
56.	558 yds. at	12s.	1¾d.	Answer, £338. 17s. 4½d.
57.	237 yds. at	14s.	10d.	Answer, £175. 15s. 6d.
58.	973 yds. at	16s.	3½d.	Answer, £792. 11s. 9½d.
59.	455 yds. at	18s.	5d.	Answer, £418. 19s. 7d.
60.	179 yds. at	19s.	11¾d.	Answer, £178. 16s. 3½d.

5th. When the price is pounds, shillings, pence, and farthings.

Example.

735 Cwt. at £3. 16s. 9¾d. per Cwt.

$$3$$

		Cwt	£
		2205 = 735 at 3	per cwt.
10s. ..	½	367 . 10	
6s. 8d..	⅓	245 . 0	
1d..	8¹₀	3 . 1 . 3	
½d..	½	1 . 10 . 7½	
¼d..	½	0 . 15 . 3¾	
		£2822 . 17 . 2¼	

Multiply by the number of pounds, and take parts for the shillings, pence, and farthings, as before.

Observe, 10s. the ½, and 6s. 8d. the ⅓ of a £, are taken out of the top line, which is the value of 735 Cwt. at £1. per Cwt. It is a common mistake to take these out of (2205) the wrong line.

Example proved.

1 Cwt. : 735 Cwt. : : £3 , 16 . 9¾d.

Or, £3 . 16 . 9¾=3687 farthings
735

18435
11061
25809

4)2709945 farthings.

12)677486 . ¼

2,0)5645,7 . 2

£2822 . 17 . 2¼

EXERCISES.

| 61. | 473 Cwt. at £1. 12s. 8d. | Answer, £772. 11s. 4d. |
| 62. | 305 Cwt. at £3. 7s. 9¼d. | Answer, £1033. 10s. 1¼d. |

63. 237 Cwt. at £4. 14s. 3¾d.   Answer, £1117. 12s. 0¾d.
64. 520 Cwt. at £7. 18s. 1½d.   Answer, £4111. 5s. 0d.
65. 319 Cwt. at £5. 11s. 10d.   Answer, £1783. 14s. 10d.
66. 195 Cwt. at £13. 16s. 10¾d.   Answer, £2699. 14s. 8¼d.

6th. When the quantity has a fraction annexed.

Example.

	lbs. 437¾		at	s. d. 14. 9½

			s. d.
10s...½	218.10	½..½	7. 4½
4s...⅕	87. 8	¼..½	3. 8¼
8d...⅙	14.11. 4		
1d...⅛	1.16. 5		11. 1
½d...½	18. 2½		
	11. 1		

£323.15. 0½

Take parts out of the quantity as before; to which add ¼, ½, ¾, &c. of the price for the Answer.

67. 372¼ gal. at   5s. 9d.   Answer, £107. 0s. 5¼d.
68. 407½ gal. at   18s. 8¾d.   Answer, £381. 12s. 1¼d.
69. 253¾ gal. at £3. 10s. 1½d.   Answer, £889. 14s. 2¼d.
70. 976⅛ gal. at £1. 7s. 5d.   Answer, £1338. 12s. 4½d.
71. 814¾ gal. at   13s. 2¾d.   Answer, £538. 18s. 5½d.
72. 705¼ gal. at £4. 1s. 8¼d.   Answer, £2880. 10s. 1¼d.

7th. When the quantity is of several denominations.

Example. 23 Cwt. 3 qrs. 17 lb. of Sugar, at £5. 18s. 5½d. per Cwt.

	£ s. d.	
	5.18. 5½ × 23	
	7	
	4 . 19. 2¾	
	3	
	124. 7. 7½	
	11.16.11	top line × by 2.
2 qrs..½	2.19. 2¾	
1 qr...½	1. 9. 7¼	
14 lb. ..½	14. 9½	
2 lb. ..⅐	2. 1¼	
1 lb. ..½	1. 0½	
	£141.11. 3¾	

Another way.

$$£ \quad s. \quad d.$$

23 Cwt. 3 qrs. 17 lb. of Sugar, at 5.18.5¼ per Cwt.

5		2 qrs...½	2.19. 2¾
———		1 qr. ..½	1. 9. 7¼
115		14 lb....½	14. 9½
10s....½	11 .10	2 lb. ....¼	2. 1½
5s....½	5 .15	1 lb. ....½	1. 0½
3s. 4d.⅙	3 .16. 8		—————
1d. .. ¼⁰	1 .11		£5. 6. 9¼
½d. ...½	11½		
	—————		
	5. 6. 9¼		

£141 .11. 3¾

Example.　　17 Cwt. 2 qrs. 15 lb. of Tea, at 7s. 9d. per lb.

4	
———	
70 qrs.	
28	
———	
1975 lb. at 7s. 9d.	The Cwt. &c. are first brought
———	into lbs., the price being 7s. 9d.
5s....... ¼ 　493 .15	per lb.
2s. 6d. ... ½ 　246 .17. 6	
3d. ...... ⅒ 　24 .13. 9	
—————	
£765. 6. 3	

73. 17½ Cwt. of Sugar, at 9¾d. per lb.　　Answer, £79. 12s. 6d.
74. 207¼ yards, at 4s. 7½d. per yard.　　Answer, £47. 18s. 6¼d.
75. 176 Cwt. 2 qrs. 10 lb. at £4. 5s. 7½d. per Cwt.
　　　　　　　　　　　　　　　　Answer, £756. 0s. 5¼d.
76. 16 Cwt. 3 qr. 17 lb. at £317. 12s. 6d. per Ton.
　　　　　　　　　　　　　　　　Answer, £268. 8s. 5d.
77. 17 hdds. 14 gal. of Wine, at 3s. 4½d. per pint.
　　　　　　　　　　　　　　　　Answer, £1464. 15s.
78. 175 oz. 15 dwts. of Silver, at 5s. 7½d. per oz.
　　　　　　　　　　　　　　　　Answer, £49. 8s. 7d.
79. 19 Cwt. 1 qr. 18 lb. at £42. 17s. 2¾d. per Ton.
　　　　　　　　　　　　　　　　Answer, £41. 12s.
80. 26 Cwt. 1 qr. 12lb. at 3s. 2d. per lb.　　Answer, £467. 8s.
81. 729 oz. 11 dwts. of Silver, at 4s. 6d. per ounce.
82. 3 qrs. 12 lb. at £2. 6s. 10½d. per Cwt.
83. 79 acres, 3 roods, 29 poles, at £8. 15s. 6d. per acre.
84. 15 Cwt. 1 qr. 19 lb. at £52. 13s. 8d. per ton.
85. 31 Cwt. 3 qrs. 7 lb. at 2s. 10¾d. per lb.
86. 79 Cwt. 0 qr. 15 lb. at £4. 17s. 7d. per Cwt.

FRACTIONS,

VULGAR OR DECIMAL.

OF VULGAR FRACTIONS.

A FRACTION is a quantity which represents a part or parts of an integer or whole.

A Vulgar Fraction is represented by two numbers placed one above the other, with a line between them, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{7}$, &c. The number above the line is called the Numerator, and that below, the Denominator.

The Denominator shews into how many parts an unit is divided; and the Numerator how many of those parts are represented by the fraction: thus, $\frac{5}{9}$ means, that an unit, as one shilling, one gallon, &c., is divided into nine parts, and five of those parts are to be taken.

Of Vulgar Fractions there are six sorts.

(1.) A SIMPLE Fraction has but one Numerator, and one Denominator, as $\frac{2}{3}$, $\frac{1}{5}$, $\frac{23}{17}$, &c.

(2.) A COMPOUND Fraction is the fraction of a fraction, and is known by the word OF placed between the parts; as $\frac{2}{3}$ of $\frac{5}{7}$; $\frac{3}{5}$ of $\frac{9}{11}$, &c.

(3.) A PROPER Fraction is when the Numerator is less than the Denominator, as $\frac{1}{2}$, $\frac{5}{7}$, $\frac{13}{19}$, &c.

(4.) An IMPROPER Fraction is when the Numerator is equal to, or greater than the Denominator, as $\frac{3}{3}$, $\frac{7}{7}$, $\frac{8}{5}$, $\frac{13}{4}$, &c.

(5.) A MIXED Number is composed of a whole number and a fraction; as $4\frac{1}{2}$, $18\frac{5}{7}$, &c.

(6) A COMPLEX Fraction has a fraction or mixed number, in either or both of its terms; as $\frac{\frac{3}{5}}{7}$, $\frac{8}{9\frac{1}{5}}$, $\frac{2\frac{1}{2}}{3\frac{1}{4}}$, &c.

REDUCTION OF VULGAR FRACTIONS.

~~~~~~~~~~~~~

CASE 1st. To find the common measure of a Vulgar Fraction.

Example.

What is the common measure of $\frac{252}{364}$?

```
252) 364 (1
     252
     ———
     112) 252 (2
          224
          ———
          28) 112 (4
              112
              ———
              · · ·
              ═══
```

RULE.

Divide the greater term by the less, and the divisor by the remainder till there is no remainder. The *last* divisor is the common measure.

Answer, 28 common measure.

EXERCISES.

1. What is the common measure of $\frac{112}{128}$?   Answer, 8.
2. What is the common measure of $\frac{345}{435}$?   Answer, 15.
3. What is the common measure of $\frac{875}{1125}$?   Answer, 125.
4. What is the common measure of $\frac{5184}{6912}$?   Answer, 1728.
5. What is the common measure of $\frac{4446}{11115}$?   Answer, 2223.

CASE 2d. To reduce a fraction to its lowest terms.

Example.

Reduce $\frac{182}{196}$ to its lowest terms.

```
182) 196 (1
     182
     ———
     14) 182 (13
         182
         ———
         · · ·
         ═══
```

RULE.

Find the common measure by Case 1st. Then divide both numerator and denominator by the common measure.

14 is the common measure :
   then 14) $\frac{182}{196}=\frac{13}{14}$.

Answer, $\frac{13}{14}$, lowest terms.*

---

* A Fraction still retains the same value, although reduced into lower terms. A Fraction ending with two even numbers, can be reduced into lower terms, being divisible by 2. Also, if one term of a Fraction end with 5, and the other with 0, or if both end with 5, the Fraction is divisible by 5.

Another Example.

Reduce $\frac{132}{240}$ to its lowest terms.

(2)   (4)   (6)

$\frac{132}{240} = \frac{66}{120} = \frac{11}{20} = \frac{1}{2}.$

Answer, $\frac{1}{2}$, lowest terms.

OR,

Divide by any number that will divide both numerator and denominator without a remainder.

EXERCISES.

6. Reduce $\frac{48}{56}$ to its lowest terms.   Answer, $\frac{6}{7}$

7. Reduce $\frac{132}{240}$ to its lowest terms.   Answer, $\frac{1}{2}$

8. Reduce $\frac{72}{84}$ to its lowest terms.   Answer, $\frac{6}{7}$

9. Reduce $\frac{90}{105}$ to its lowest terms.   Answer, $\frac{12}{7}$

10. Reduce $\frac{144}{192}$ to its lowest terms.   Answer, $\frac{23}{8}$

11. Reduce $\frac{7631}{8415}$ to its lowest terms.   Answer, $\frac{13}{15}$

CASE 3d. To reduce a mixed number to an improper fraction.*

Example.

Reduce $8\frac{5}{7}$ to an improper fraction.

     8

     7 Denominator.

     ——

     56

     5 Numerator.

     ——

     61

     ——

     7

     ==

Answer, $\frac{61}{7}$, improper fraction.

RULE.

Multiply the whole number into the denominator of the fraction, and add the numerator to the product for a new numerator, under which place the given denominator.

EXERCISES.

12. Reduce $4\frac{3}{4}$ to an improper fraction.   Answer, $\frac{19}{4}$

13. Reduce $22\frac{1}{5}$ to an improper fraction.   Answer, $\frac{111}{5}$

14. Reduce $29\frac{7}{16}$ to an improper fraction.   Answer, $\frac{471}{16}$

15. Reduce $100\frac{1}{8}$ to an improper fraction.   Answer, $\frac{5019}{50}$

16. Reduce $79\frac{1}{2}$ to an improper fraction.   Answer, $\frac{1443}{18}$

17. Reduce $514\frac{5}{16}$ to an improper fraction.   Answer, $\frac{8229}{16}$

* To express a whole number like a fraction, put 1 for the denominator, thus—
7 as $\frac{7}{1}$, 13 as $\frac{13}{1}$.

CASE 4th.   To reduce an improper fraction to its proper terms.

Example.	RULE.
Reduce $\frac{61}{7}$ to proper terms. $7)\dfrac{61}{8 \cdot \frac{5}{7}}$ $=$ Answer, $8\frac{5}{7}$, proper terms.	Divide the numerator by the denominator: the quotient will be the whole number; and if there be any remainder, place it over the denominator for a fraction.

Case 3d and 4th prove each other.

EXERCISES.

18. Reduce $\frac{19}{8}$ to proper terms.   Answer, $2\frac{3}{8}$.

19. Reduce $\frac{141}{13}$ to proper terms.   Answer, $10\frac{11}{13}$.

20. Reduce $\frac{360}{15}$ to proper terms.   Answer,   24.

21. Reduce $\frac{807}{14}$ to proper terms.   Answer, $57\frac{9}{14}$.

22. Reduce $\frac{1035}{23}$ to proper terms.   Answer, $44\frac{13}{23}$.

23. Reduce $\frac{7118}{35}$ to proper terms.   Answer, $206\frac{8}{35}$.

24. Reduce $\frac{8229}{16}$ to proper terms.   Answer, $514\frac{5}{16}$.

CASE 5th.   To reduce a compound fraction to a simple one.

Example.	RULE.
Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{7}$ to a simple fraction. $\frac{1}{2} \times \frac{3}{4} \times \frac{5}{7} = \frac{15}{56}$. Answer, $\frac{15}{56}$.	Multiply all the numerators together for a numerator — and all the denominators for a denominator.
Example. Reduce $\frac{3}{5}$ of $8\frac{1}{2}$ to a simple fraction. $8\frac{1}{2} = \frac{17}{2}$ then $\frac{3}{5} \times \frac{17}{2} = \frac{51}{10} = \frac{51}{10}$. Answer, $\frac{51}{10}$.	If any of the proposed quantities be whole or mixed numbers, first reduce them to improper fractions, as in Case 3d.

Example.

Reduce ⅔ of ¾ of $\frac{4}{11}$ to a simple fraction.

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{11} = \frac{4}{11}$$

Answer, $\frac{4}{11}$.

When the same number is found in a numerator and denominator, it may be struck out of each. Or, when a numerator and denominator can be divided by any number, the quotients may be used instead.*

EXERCISES.

25. Reduce ⅔ of ¾ to a simple fraction.   Answer, ½

26. Reduce $\frac{4}{7}$ of ⅚ to a simple fraction.   Answer, $\frac{2\cdot5}{3\cdot5}$

27. Reduce ⅔ of ½ of ¼ to a simple fraction.   Answer, ¼

28. Reduce ⅘ of 8¼ to a simple fraction.   Answer, $\frac{3\cdot3}{5}$

29. Reduce $\frac{5}{6}$ of ¼ of 12½ to a simple fraction.   Answer, $\frac{3\cdot5}{1\cdot8}$

30. Reduce ⅓ of ⅘ of 3¼ to a simple fraction.   Answer, $\frac{5\cdot2}{4\cdot1}$

31. Reduce ½ of $\frac{9}{13}$ of 1¼ to a simple fraction.   Answer, $\frac{6\cdot8}{2\cdot6\cdot7}$

CASE 6th.   To find the least common multiple of any given numbers.

Example.

Find the least common multiple of 4, 6, 8, and 12.

$$
\begin{array}{l}
4)\ 4\ .\ 6\ .\ 8\ .\ 12 \\
\hline
2)\ 1\ .\ 6\ .\ 2\ .\ 3 \\
\hline
3)\ 1\ .\ 3\ .\ 1\ .\ 3 \\
\hline
\quad 1\ .\ 1\ .\ 1\ .\ 1
\end{array}
$$

The divisors are 4, 2, and 3, consequently $4 \times 2 \times 3 = 24$.

Answer, 24, least com. multiple.

RULE.

Place the given numbers in a line, and divide by any number that will divide two or more of them without a remainder; put the quotients beneath, and bring down all numbers that could not be divided. The product of all the divisors is the least common multiple.

EXERCISES.

32. Required the least common multiple of 3, 4, and 8,
                                                    Answer, 24.

33. Required the least common multiple of 2, 3, 4, 5, and 6.
                                                    Answer, 60.

---

* This method of cancelling is very useful, as it shortens the work, and brings out the fraction in the lowest terms.

34. Required the least common multiple of 10, 18, 30, and 45.

Answer, 90.

35. Required the least common multiple of 12, 6, 9, and 8.

Answer, 72.

CASE 7th. To reduce fractions of different denominators to equivalent fractions having a common denominator.

Example.

Reduce $\frac{2}{3}$, $\frac{5}{8}$, and $\frac{7}{9}$ to fractions having a common denominator.

$$2 \times 8 \times 9 = 144$$
$$5 \times 3 \times 9 = 135$$
$$7 \times 3 \times 8 = 168$$

New numerators.

$3 \times 8 \times 9 = 216$ Common denominator.

Answer, $\frac{144}{216}$, $\frac{135}{216}$, and $\frac{168}{216}$.*

Example.

Reduce 7, $3\frac{1}{4}$, and $\frac{2}{3}$ of $\frac{3}{5}$, to fractions having a common denominator.

First, $7 = \frac{7}{1}$; $3\frac{1}{4} = \frac{13}{4}$; & $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$

The fractions are $\frac{7}{1}$, $\frac{13}{4}$, and $\frac{2}{5}$.

$$7 \times 4 \times 5 = 140$$
$$13 \times 1 \times 5 = 66$$
$$2 \times 1 \times 4 = 8$$

New numerators.

$1 \times 4 \times 5 = 20$ Common denominator.

Answer, $\frac{140}{20}$, $\frac{65}{20}$, and $\frac{8}{20}$.

RULE.

Multiply each numerator into all the denominators, except its own, for new numerators; and multiply all the denominators together for a common denominator.

Should any of the proposed quantities be whole or mixed numbers, or compound fractions, reduce them to simple fractions, and proceed as above.

EXERCISES.

36. Reduce $\frac{1}{4}$ and $\frac{3}{5}$ to fractions having a common denominator.

Answer, $\frac{5}{20}$, $\frac{12}{20}$.

37. Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$ to fractions having a common denominator.

Answer, $\frac{12}{24}$, $\frac{18}{24}$, $\frac{20}{24}$.

38. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{7}$ to fractions having a common denominator.

Answer, $\frac{63}{105}$, $\frac{70}{105}$, $\frac{90}{105}$.

* It will appear, by reducing the new fractions into the lowest terms, that the values are not changed: thus $\frac{144}{216} = \frac{2}{3}$; $\frac{135}{216} = \frac{5}{8}$; and $\frac{168}{216} = \frac{7}{9}$.

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39. Reduce ⅓ of 5, and ⁷⁄₉ to fractions having a common denominator.   Answer, ⁴⁵⁄₉, ⁷⁄₉.

40. Reduce ⅘, ⅞, and ⅝ to fractions having a common denominator.   Answer, ²⁵⁶⁄₃₂₀, ²⁸⁰⁄₃₂₀, ²⁰⁰⁄₃₂₀.

41. Reduce ⅓ of ⅖, 5¼, and ⅚ to fractions having a common denominator.   Answer, ⁸⁴⁄₇₆₀, ¹⁴⁷⁹⁄₇₆₀, ⁴⁰⁰⁄₇₆₀.

42. Reduce ⅝, ₇⁄₁₁, ⅘, and ½ to fractions having a common denominator.   Answer, ⁸¹⁴⁄₁₃₂₀, ⁸⁴²⁄₁₃₂₀, ¹¹⁵⁸⁄₁₃₂₀, ⁶⁹³⁄₁₃₂₀.

CASE 8th. To reduce fractions to others that shall have the least common denominator.

Example.

Reduce ⅓, ₇⁄₁₀, and ¹¹⁄₁₂ to the least common denominator.

    2) 4 . 10 . 12
    ─────────────
    2) 2 . 5 . 6
    ─────────────
       1 . 5 . 3
    ─────────────

2×2×5×3=60 least com. multiple.

then ⁶⁰⁄₄=15.   15× 3=45 ⎫
     ⁶⁰⁄₁₀= 6.   6× 7=42 ⎬ New
     ⁶⁰⁄₁₂= 5.   5×11=55 ⎭ numerators.

Answer, ⁴⁵⁄₆₀, ⁴²⁄₆₀, ⁵⁵⁄₆₀.

RULE.

Find the least common multiple of all the denominators, by Case 6th, for the new denominator.

Divide this by each of the given denominators; and multiply the quotient by the respective numerators for the new numerator.

EXERCISES.

43. Reduce ½, ⅔, and ⅚ to the least common denominator.
                                Answer, ³⁄₆, ⁴⁄₆, & ⁵⁄₆.

44. Reduce ⅓, ⅔, ¾, and ⅚ to the least common denominator.
                Answer, ₁⁄₁₂, ₁⁄₁₂, ₁⁄₁₂, & ¹⁰⁄₁₂.

45. Reduce ⅔, ⅘, ⅚, and ₇⁄₁₀ to the least common denominator.
                Answer, ⁴⁰⁄₆₀, ⁴⁸⁄₆₀, ⁵⁰⁄₆₀, & ⁴²⁄₆₀.

46. Reduce ⅗, ½, ⅐, and ⅚ to the least common denominator.
                Answer, ¹²⁶⁄₂₁₀, ¹⁰⁵⁄₂₁₀, ³⁰⁄₂₁₀, & ¹⁷⁵⁄₂₁₀.

47. Reduce ¾, ⅚, ₄⁄₁₁, and ¹¹⁄₁₂ to the least common denominator.
                Answer, ²⁷⁄₃₆, ³⁰⁄₃₆, ¹¹⁄₃₆, & ¹¹⁄₃₆.

48. Reduce ¹¹⁄₄, ¹⁵⁄₈, ¹¹⁄₂, and ³¹⁄₆ to the least common denominator.
                Answer, ²⁶⁶⁄₂₄₈, ²⁴⁴⁄₂₄₈, ⁶⁶⁄₂₄₈, & ³⁷⁴⁄₂₄₈.

CASE 9th. To reduce a complex fraction to a simple one.

Example.

Reduce $\dfrac{3\frac{1}{4}}{6\frac{1}{3}}$ to a simple fraction.

$$\frac{3\frac{1}{4}}{6\frac{1}{3}} = \frac{\frac{13}{4}}{\frac{19}{3}}$$

(extr.)
then $\dfrac{13 \times 3}{4 \times 19} = \frac{39}{76}$
(mea.)

Answer, $\frac{39}{76}$

RULE.

Make the numerator and the denominator improper fractions; then, multiply the extremes for a numerator, and the means for a denominator.

Another Example.

Reduce $\dfrac{41}{73\frac{1}{4}}$ to a simple fraction.

$$\frac{41}{73\frac{1}{4}} = \frac{\frac{41}{1}}{\frac{293}{4}}$$

then, $\dfrac{41 \times 4}{293 \times 1} = \frac{164}{293}$

Answer, $\frac{164}{293}$

EXERCISES.

49. Reduce $\dfrac{2\frac{1}{3}}{3\frac{1}{4}}$ to a simple fraction. Answer, $\frac{28}{39}$

50. Reduce $\dfrac{7\frac{1}{2}}{9\frac{1}{4}}$ to a simple fraction. Answer, $\frac{30}{37}$

51. Reduce $\dfrac{4}{7\frac{1}{2}}$ to a simple fraction. Answer, $\frac{8}{15}$

52. Reduce $\dfrac{9\frac{1}{3}}{15}$ to a simple fraction. Answer, $\frac{28}{45}$

53. Reduce $\dfrac{47}{65\frac{1}{4}}$ to a simple fraction. Answer, $\frac{4}{7}$

54. Reduce $\dfrac{20\frac{1}{9}}{22\frac{1}{11}}$ to a simple fraction. Answer, $\frac{221}{243}$

CASE 10th. To find the value of a fraction in the known parts of the integer.

Example.

Required the value of ¼ of a pound sterling.

$$\begin{array}{r} 5 \\ 20 \text{ shillings} = \pounds 1. \\ \hline 9)\overline{100} \\ \hline 11 . 1 \\ 12 \text{ pence} = 1 \text{ sg.} \\ \hline 9)\overline{12} \\ \hline 1 . 3 \\ 4 \text{ fgs.} = 1d. \\ \hline 9)\overline{12} \\ \hline 1 \ \tfrac{3}{4} = \tfrac{1}{4}. \end{array}$$

Answer, 11s. 1¼d.+¼.

RULE.

Multiply the numerator by the next lower denomination, and divide by the denominator; if any thing remain, multiply it by the next lower denomination, and divide as before, and so proceed to the lowest denomination; and the several quotients will be the value required.

EXERCISES.

55. What is the value of ¾ of a shilling?   Answer, 10½d.

56. Required the value of ¼ of a guinea.   Answer, 4s. 8d.

57. Reduce ⅜ of a yard of Cloth to its proper quantity.
                                Answer, 3 qrs. 2 nls.

58. Reduce ¼ of a pound avoirdupois to its proper quantity.
                                Answer, 8 oz. 14 drs. ⅛

59. What is the proper quantity of ¾ of a hogshead of Beer?
                                Answer, 36 gallons.

60 Reduce ¼ of a pound troy to its proper quantity.
                                Answer, 9 oz. 12 dwts.

61. Reduce $\tfrac{7}{13}$ of a day to its proper quantity.
                        Answer, 12 hrs. 55 min. 23 sec. $\tfrac{1}{13}$

62. What is the value of $\tfrac{8}{13}$ of a pound sterling?—Answer, 10s. 8d.

63. Reduce ½ of a mile to its proper quantity.
                        Answer, 4 fur. 22 poles. 4 yds. 2 feet, 1 in. ½

64. Reduce $\tfrac{17}{153}$ of a ton to its proper quantity.
                        Answer, 3 Cwt. 1 qr. 18 lb. 12 oz. 8 dr. $\tfrac{72}{7}$.

CASE 11th. To reduce any given quantity to the fraction of a higher denomination, retaining the same value.

Example.

Reduce 3 Cwt. 2 qrs. 8 lb. to the fraction of a ton.

$$3 \text{ Cwt. 2 qrs. 8 lb.}$$
$$4$$
_____
$$14 \text{ qrs.}$$
$$28$$
_____
$$400 \text{ lb.}$$
_____
$$2240 \text{ lb.} = 1 \text{ ton.}$$

Answer, $\frac{400}{2240} = \frac{10}{56} = \frac{5}{28}$

RULE.

Reduce the given quantity to the lowest term mentioned for a numerator and place for a denominator as many of the small as make one of the greater denomination.*

Cases 10 and 11 prove each other.

EXERCISES.

65. Reduce 18 shillings to the fraction of a pound.   Answer, $\frac{9}{10}$.

66. Reduce 7s. 3d. to the fraction of a pound.   Answer, $\frac{29}{80}$.

67. Reduce 8 oz. 2 dwts. to the fraction of a pound troy.   Ans. $\frac{27}{48}$.

68. Reduce 2 qrs. 2 nls. to the fraction of a yard of cloth.   Ans. $\frac{5}{8}$.

69. Reduce 4s. 8d. to the fraction of a guinea.   Answer, $\frac{2}{9}$.

70. Reduce 2 roods, 20 poles to the fraction of an acre.   Ans. $\frac{5}{8}$.

71. Reduce 12 gallons of beer to the fraction of a hhd.   Ans. $\frac{2}{9}$.

72. Reduce 2 weeks, 5 days, to the fraction of a month.   Ans. $\frac{19}{30}$.

73. Reduce 2 qrs. 14 lb. 1 oz. to the fraction of a Cwt.   Ans. $\frac{1121}{1792}$.

74. Reduce 15s. 9¾d. to the fraction of a pound.   Answer, $\frac{253}{320}$.

75. Reduce 13s. 8¼d. to the fraction of a guinea.   Answer, $\frac{73}{112}$.

CASE 12th. To reduce a fraction from one denomination to that of another, retaining the same value.

Example.

Reduce $\frac{3}{4}$ of a pennyweight to the fraction of a pound troy.

$$\frac{3}{5 \times 20 \times \frac{12}{4}} = \frac{1}{5 \times 20 \times 4} = \frac{1}{400}$$

Answer, $\frac{1}{400}$

RULE.

When it is from a less name to a greater, multiply the *denominator* by all the denominations from that given, to the one sought.

* Some Arithmeticians have given another Rule for reducing the fractions; but the above may be best understood by the learner.

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Example.

Reduce ⅔ of a pound to the fraction of a farthing.

$$\frac{2 \times 20 \times \cancel{12}^{4} \times 4}{\cancel{9}_{3}} =$$

$$\frac{2 \times 20 \times 4 \times 4}{3} = {}^{6}\tfrac{4}{3}{}^{0}$$

Answer, ⁶⅓⁰

If it be from a greater name to a less, multiply the *numerator* by all the denominations from that given to the one sought.

EXERCISES.

76. Reduce ⅛ of a penny to the fraction of a pound.   Answer, ₂₁₆

77. Reduce ⅔ of a farthing to the fraction of a pound.   Ans. ₁₄₄₀

78. Reduce ⅛ of a pound to the fraction of a penny.   Ans. ⁴⁸⁰⁄₃

79. Reduce ₁⁵₇ of a guinea to the fraction of a penny.   Ans. ¹²⁴⁵⁰

80. Reduce ⁴⁄₉ of a lb. avoir. to the fraction of a Cwt.   Ans. ₃₁₅₂

81. Reduce ₁₁₂ of a Cwt. to the fraction of a pound.   Ans. ⅔

82. Reduce ₅⁷₁ of a week to the fraction of an hour.   Ans. ¹¹₇₇⁶

83. Reduce ⅞ of a nail to the fraction of a yard.   Answer, ₁⁷₄₇

84. Reduce ₁⁵₂ of a mile to the fraction of a foot.   Ans. ²²⁰⁰⁄₁

85. Reduce ₁⁵₃ of a pint of wine to the fraction of a hhd.   Ans. ₇₁₈

86. Reduce ₁₁⁄ of a pound to the fraction of a farthing.   Ans. ⁵¹⁵⁰⁄₇

ADDITION OF VULGAR FRACTIONS.

Example.

Add ₁³₁, ₁⁵₁, and ₁²₁ together.
$$3+5+2=10$$

Answer, ₁¹⁰₁

RULES.

1. When the fractions have a common denominator, add the numerators together, and place the common denominator under their sum.

Example.

Add ¾, ⅚, and ⅞ together.
$$\left.\begin{array}{l}3\times6\times9=162\\5\times4\times9=180\\8\times4\times6=192\end{array}\right\} \begin{array}{l}\text{New}\\\text{numerators.}\end{array}$$

$$\overline{534}\text{ Total.}$$

$$4\times6\times9=216\text{ Com. denom.}$$

6) ₂⁵₁³₆⁴ = ₃⁹⁰₆ = 2₃⁷₆ Answer.

2. When the fractions have not a common denominator, reduce them to one, by Case 7th, and add the numerators together, as before.

Example.

What is the sum of $\frac{5}{8}$, $\frac{3}{5}$ of $\frac{4}{7}$, and $\frac{2}{3}$ of 3 ?

$$\frac{3}{5} \text{ of } \frac{4}{7} = \frac{12}{35} \qquad \frac{2}{\underset{3}{\cancel{9}}} \text{ of } \frac{3}{1} = \frac{2}{3}$$

The fractions to be added are

$$\frac{5}{8}, \quad \frac{12}{35}, \quad \frac{2}{3}$$

$$\left. \begin{array}{l} 5 \times 35 \times\ \ 3 = 525 \\ 12 \times\ \ 8 \times\ \ 3 = 288 \\ 2 \times\ \ 8 \times 35 = 560 \end{array} \right\} \begin{array}{l} \text{New} \\ \text{numerators.} \end{array}$$

$\overline{\quad 1373 \quad}$ Total.

$8 \times 35 \times 3 = 840$ Com. denom.

Answer, $\frac{1373}{840} = 1\frac{533}{840}$

Example.

Add $47\frac{3}{4}$, $29\frac{5}{7}$, and $35\frac{9}{14}$ together.

The fractions are $\frac{3}{4}$, $\frac{5}{7}$, and $\frac{9}{14}$.

	Whole numbers.
$3 \times 7 \times 14 = 294$	
$5 \times 4 \times 14 = 280$	47
$9 \times 4 \times\ \ 7 = 252$	29
$\overline{\qquad 826 \qquad}$	35
	$2\frac{3}{28}$
$4 \times 7 \times 14 = 392$	$\overline{\quad 113\frac{3}{28} \quad}$

$$14) \frac{826}{392} = \frac{59}{28} = 2\frac{3}{28}$$

Answer, $113\frac{3}{28}$.

EXERCISES.

87. Required the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{4}$.   Answer, $1\frac{1}{2}$

88. What is the sum of $\frac{2}{3}$, $\frac{1}{7}$, and $\frac{1}{9}$?   Answer, $1\frac{1}{9}$

89. What is the sum of $\frac{1}{2}$ and $\frac{3}{4}$?   Answer, $1\frac{1}{4}$

90. Add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{2}$ together.   Answer, $1\frac{1}{12}$

91. Required the sum of $\frac{4}{5}$, $\frac{11}{15}$, and $\frac{3}{5}$.   Answer, 2.

92. Add $\frac{3}{7}$ and $\frac{4}{5}$ together.   Answer, $1\frac{9}{35}$

93. What is the sum of $\frac{2}{5}$ and $9\frac{1}{4}$?   Answer, $9\frac{13}{20}$

94. Add $\frac{1}{4}$ of $\frac{4}{5}$, $\frac{3}{3}$ of $\frac{6}{7}$, and $\frac{1}{2}$ together.   Answer, $1\frac{47}{70}$

95. Required the sum of 74¼ and 274⅞.   Answer, 349⅝

96. Required the sum of 7¼ and 9¾.   Answer, 16¹¹⁄₁₂

97. Add ⅞, 4½, and ½ of ¼ together.   Answer, 5⅞₅

98. What is the sum of ½ of ⅔, ¼ of 19, and ⅔ of 12?   Answer, 18⁴⁰₅₁

99. Required the sum of 100⅞, 2000¾, and 1764¼.   Ans. 3866¹³₅₆

Example.

What is the sum of ⅞ of a guinea, ¼ of a shilling, and ½ of a penny?

	s.	d.
⅞ of a guinea .. =	9	0
¼ of a shilling.. =	0	4½
½ of a penny .. =	0	0½ ½
Answer ..	9	5 ½

5. When there are fractions of the integers of money, weight, measures, &c., find their values by Case 10th, and add as in Compound Addition.

EXERCISES.

100. Required the sum of ½ of a pound, and ¼ of a shilling.
        Answer, 11s. 5¾d. ½

101. What is the amount of ¾ of a guinea, ⅙ of a shilling, and ½ of a penny?   Answer, 14s. 7½d. ⅔₇

102. Add together ¼ of a week, ½ of a day, and ⅓ of an hour.
        Answer, 2 days, 2 hrs. 12 min.

103. What is the sum of ½ of a hogshead of beer, and ⅞ of a barrel?
        Answer, 62 gals. ½ pt.

104. Required the sum of ⅔ of a yard, ¾ of a foot, and ⅔ of an inch.
        Answer, 2 ft. 9 in. 2 b. c.

105. Add ₇⁹₀ of a Cwt., ⅙ of a quarter, and ₁₁⁹ of a lb. together.
        Answer, 1 qr. 13 lb. 13 oz. 1 dr. ₁₁⁹

106. What is the sum of ½ of a lb. troy, ½ of an ounce, and ₁₀⁷ of a pennyweight?   Answer, 5 oz. 8 dwts. 11 gis. ₁₅⁷

107. Required the sum of ⅔ of £15—3⅔ of a £—½ of ⅚ of ¾ of a £— and ⅔ of ⅚ of a shilling.   Answer, £7. 17s. 5d. ⅘

SUBTRACTION OF VULGAR FRACTIONS.

Examples.

From $\frac{8}{11}$ take $\frac{3}{7}$

$8\times\ 7=56$ } New numerators.
$3\times11=33$ }

---

23 Difference.

$11\times\ 7=77$ Com. denominator.

Answer, $\frac{23}{77}$

From $2\frac{1}{4}$ take $\frac{1}{3}$ of $\frac{7}{8}$

$2\frac{1}{4}=\frac{9}{4}$ $\frac{1}{3}$ of $\frac{7}{8}=\frac{7}{72}$

The fractions are, $\frac{9}{4}$, $\frac{7}{72}$

$9\times72=648$ } New numerators.
$7\times\ 4=\ 28$ }

---

620 Difference.

$4\times72=288$ Com. denominator.

4) $\frac{620}{288}=\frac{155}{72}=2\frac{11}{72}$ Answer.

Rule.

Prepare the fractions, when necessary, as shewn in the examples in Addition.

Find a common denominator by Case 7th; then subtract the less numerator from the greater, under which place the common denominator.

Exercises.

108. From $\frac{5}{7}$ take $\frac{2}{7}$ Answer, $\frac{3}{7}$

109. From 1 take $\frac{11}{12}$ Answer, $\frac{1}{12}$

110. What is the difference between $1\frac{1}{2}$ and $\frac{4}{5}$?   Answer, $\frac{32}{54}$

111. Required the difference between $\frac{1}{4}$ and $\frac{2}{3}$ Answer, $\frac{5}{12}$

112. From $5\frac{3}{4}$ take $\frac{9}{10}$ Answer, $4\frac{17}{20}$

113. From $2\frac{1}{4}$ take $\frac{4}{5}$ Answer, $1\frac{9}{20}$

114. What is the difference between 25 and $21\frac{5}{6}$ Answer, $3\frac{1}{6}$

115. What is the difference between 138 and $14\frac{3}{11}$? Answer, $123\frac{8}{11}$

116. Required the difference between $9\frac{3}{4}$ and $4\frac{7}{8}$ Answer, $4\frac{7}{8}$

117. From $36\frac{9}{11}$ take $16\frac{2}{3}$ Answer, $20\frac{3}{33}$

118. From $\frac{3}{4}$ of $\frac{11}{12}$, take $\frac{1}{2}$ of $\frac{2}{3}$ Answer, $\frac{17}{72}$

119. From $\frac{4}{5}$ of $\frac{1}{2}$ take $\frac{3}{11}$ of $\frac{1}{2}$ Answer, $\frac{19}{165}$

120. From $\frac{1}{2}$ of a shilling take $\frac{1}{4}$ of a penny.   Answer, $5\frac{1}{4}$d.

121. From $\frac{4}{5}$ of a pound take $\frac{2}{3}$ of a shilling.   Answer, 7s. $3\frac{1}{4}$d. $\frac{1}{5}$

122. What is the difference between £8, and $\frac{5}{8}$ of a £.?

Answer, £7. 8s. 10$\frac{1}{2}$d. $\frac{2}{3}$

123. What is the difference between $\frac{1}{8}$ of a £, and $\frac{4}{5}$ of a shilling?

Answer, 3s. 4$\frac{1}{2}$d.

124. Required the difference between $\frac{3}{4}$ of a ton, and $\frac{5}{8}$ of a ton.

Answer, 2 Cwt. 2 qr.

125. Required the difference between $\frac{4}{5}$ of a ton, and $\frac{5}{9}$ of a lb.

Answer, 15 Cwt. 3 qrs. 27 lb. 2 oz. 10 drs. $\frac{2}{3}$

126. From $\frac{1}{2}$ of an ounce take $\frac{7}{8}$ of a pennyweight.

Answer, 11 dwts. 3 grs.

127. From $\frac{4}{5}$ of a guinea take $\frac{7}{16}$ of a £.   Answer, 2s. 3d.

128. From $\frac{2}{3}$ of a league take $\frac{7}{10}$ of a mile.

Answer, 1 mil. 2 fur. 16 poles.

129. From $\frac{3}{4}$ of a ton take $\frac{2}{5}$ of a Cwt.

Answer, 6 Cwt. 3 qrs. 16 lb. 12 oz. 12 drs. $\frac{4}{5}$

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MULTIPLICATION OF VULGAR FRACTIONS.

Examples.

Multiply $\frac{2}{7}$ by $\frac{5}{9}$

$$\frac{2}{7} \times \frac{5}{9} = \frac{10}{63}$$

Answer, $\frac{10}{63}$

Multiply $\frac{1}{3}$ of $\frac{4}{5}$ by $\frac{7}{10}$ of $\frac{11}{12}$

$\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$ $\frac{7}{10}$ of $\frac{11}{12} = \frac{77}{120}$

consequently $\frac{4}{15} \times \dfrac{77}{\underset{30}{120}} = \frac{77}{450}$

Answer, $\frac{77}{450}$

Required the product of $\frac{5}{9}$, $\frac{3}{7}$, and 12$\frac{1}{4}$

$$12\frac{1}{4} = \frac{49}{4}$$

$$\frac{5}{9} \times \frac{3}{7} \times \frac{\overset{7}{49}}{4} = \frac{35}{12} = 2\frac{11}{12}$$

Answer, 2$\frac{11}{12}$

RULE.

Prepare the fractions, when necessary, as in the former rules.

Then multiply the numerators together for a new numerator, and the denominators for a new denominator.

Note. Cancel the numerators and denominators, if possible, as in reducing a compound fraction to a simple one.—Case 5th.

<div align="center">EXERCISES.</div>

130. Multiply $\frac{1}{4}$ by $\frac{1}{8}$. Answer, $\frac{1}{32}$.

131. Multiply $\frac{3}{11}$ by $\frac{1}{8}$ Answer, $\frac{3}{88}$.

132. Multiply $\frac{2}{3}$ by $\frac{1}{8}$ Answer, $\frac{1}{18}$.

133. Multiply $\frac{1}{2}$ by $4\frac{1}{2}$ Answer, $2\frac{1}{4}$.

134. Multiply 57 by $\frac{9}{11}$ Answer, $46\frac{7}{11}$.

135. Multiply $7\frac{2}{5}$ by 35. Answer, 259.

136. Required the product of $7\frac{1}{4}$ and $23\frac{1}{4}$. Answer, $168\frac{9}{16}$.

137. Required the product of $\frac{3}{7}$, $\frac{4}{5}$, and $\frac{11}{12}$. Answer, $\frac{9}{15}$.

138. Required the product of $\frac{1}{4}$ of $\frac{7}{8}$, and $\frac{2}{3}$ of $\frac{1}{2}$ Answer, $\frac{9}{15}$.

139. Required the product of $\frac{1}{4}$ of $\frac{2}{3}$, and $\frac{5}{6}$ of $3\frac{2}{3}$. Answer, $\frac{11}{12}$.

140. Required the product of $85\frac{1}{4}$ by $9\frac{1}{2}$ Answer, $793\frac{11}{12}$.

141. Required the product of $\frac{7}{8}$, $\frac{4}{5}$, and $3\frac{1}{2}$ Answer, $2\frac{9}{10}$.

142. Required the product of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{2}{3}$ Answer, $\frac{1}{6}$.

143. Required the product of $\frac{7}{8}$, $\frac{2}{3}$, $5\frac{1}{4}$, and 6. Answer, $16\frac{1}{10}$.

144. Required the product of $14\frac{2}{3}$, $2\frac{1}{2}$, and $\frac{1}{2}$ of $4\frac{1}{4}$ Answer, $51\frac{17}{24}$.

145. Required the product of $821\frac{9}{11}$ by $8\frac{5}{7}$ Answer, $7161\frac{13}{77}$.

146. Required the product of 5, $4\frac{1}{4}$, $\frac{2}{3}$, and $\frac{2}{7}$ of $\frac{3}{4}$ Answer, $2\frac{5}{11}$.

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DIVISION OF VULGAR FRACTIONS.

Examples.

Divide $\frac{2}{3}$ by $\frac{4}{5}$

The terms of the divisor inverted are $\frac{5}{4}$;

consequently $\frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = 1\frac{7}{12}$

Answer, $1\frac{7}{12}$

Divide $3\frac{1}{4}$ by $9\frac{1}{2}$

$$3\frac{1}{4} = \frac{13}{4} \qquad 9\frac{1}{2} = \frac{19}{2}$$

The divisor inverted is $\frac{2}{19}$

consequently $\dfrac{\overset{13}{\cancel{13}}}{\underset{2}{\cancel{4}}} \times \dfrac{\cancel{2}}{\cancel{19}} = \frac{1}{3}$

Answer, $\frac{1}{3}$

RULE.

Prepare the fractions, when necessary, as in the former Rules.

Then invert the terms of the divisor, and proceed as in Multiplication.

Another Example.

Divide 15½ by ⅞ of ₁³₁

$$15\tfrac{1}{2} = \tfrac{140}{9} \qquad \tfrac{7}{8} \times \tfrac{3}{11} = \tfrac{21}{88}$$

The divisor inverted is $\tfrac{88}{21}$

consequently $\dfrac{\overset{20}{\cancel{140}}}{9} \times \dfrac{88}{\underset{3}{\cancel{21}}} = \dfrac{1760}{27} = 65\tfrac{5}{27}$

Answer, $65\tfrac{5}{27}$.

Exercises.

147. Divide ⅜ by ⅘	Answer, ⅜
148. Divide ½ by ₁⁷₃	Answer, 1³⁄₇
149. Divide ⁹⁄₂ by ½½	Answer, 5½½
150. Divide 9 by ⅔	Answer, 13½
151. Divide ½⅘ by 6.	Answer, ₁⁷₈
152. Divide 97 by 5½	Answer, 17½⁴⁄₉
153. Divide 15½ by 9½	Answer, 1 ½³½
154. Divide ₁⁷₁ of 8 by ₁⁹₃	Answer, 22³⁄₃₃
155. Divide ¼ of 19 by ⅔ of ⅘	Answer, 7¾
156. Divide 12½ by 3½	Answer, 3½
157. Divide 4⅘ by ⅔	Answer, 6½½
158. Divide ½ of 50 by 4½	Answer, 9⅘⁄₃
159. Divide ⅔ of ½ by ¾ of ½½	Answer, ½³⁴⁄₅
160. Divide ¾ of ½ by ½ of 7¾	Answer, ₁⁷₁ᵣ
161. Divide ⅞ of ⅔ of 5 by ½ of ¼	Answer, 15₁⁵ᵣ

RULE OF THREE IN VULGAR FRACTIONS.

Prepare the fractions, if necessary.

State the question, &c. as directed in the Rule of Three.

Multiply the second and third terms together, and also the first, with its parts inverted; and the product will be the answer, in the same denomination as the third term was left in.

Examples.

If ⅝ of a Cwt. of Sugar cost £2⅔,
what will 5¼ lb. cost?

⅝ of a Cwt. is ⅝ of ¹¹²lb. = ⁵⁶⁰⁄₉

£2⅔ = £⁸⁄₃

5¼ lb. = ²¹⁄₄ lb.

$$\overset{\text{lb.}}{\underset{9}{560}} \;.\; \overset{\text{lb.}}{\underset{4}{21}} :: \overset{£}{\frac{8}{3}}$$

$$\frac{\overset{7}{\cancel{21}}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{3}} \times \frac{9}{\cancel{560}} =$$

again, $\dfrac{\cancel{7}}{1} \times \dfrac{2}{1} \times \dfrac{9}{\underset{80}{\cancel{560}}} =$

again, $\dfrac{1}{1} \times \dfrac{\cancel{2}}{1} \times \dfrac{9}{\underset{40}{\cancel{80}}} = \overset{£}{\underset{40}{\frac{9}{1}}}$

£9
20
—————
4,0) 18,0
—————
4s. 6d.

Here the fractions are pre-
pared: ⅝ of a Cwt. is reduced
into lbs., the same denomination
as the other term. The rest are
made improper fractions.

The stating will appear from
the Rule of Proportion.

In multiplying, the numera-
tors and denominators are can-
celled, as directed in Multiplica-
tion.

The product ₉⁄₄₀ is in the same
denomination as the 3d term was
left in—pounds. This product
is reduced to its proper value.

Answer, 4s. 6d.

If ⅞ of an ell cost £⅔, what cost 8½ ells?

8½ = ¹⁷⁄₂ ells.

$$\overset{\text{Ells.}}{\tfrac{7}{8}} \quad : \quad \overset{\text{Ells.}}{\cancel{\tfrac{17}{2}}} \quad :: \quad \overset{£}{\tfrac{2}{3}}$$

$$\cancel{\tfrac{17}{2}} \times \tfrac{2}{3} \times \tfrac{8}{7} = \cancel{\tfrac{17}{7}} £$$

63) 400 (6
———
.22
20
———
63) 440 (6
———
.62
12
———
63) 744 (11
——
.51
4
———
63) 204 (⅔
———
.15 = 1½⁄₂ = ₇⁄₁

Answer, £6. 6s. 11¾d. ₇⁄₁

EXERCISES.

162. If $\frac{1}{2}$ of an ounce cost $\frac{1}{12}$ of a £, what cost 1 ounce?

Answer, £1. 5s. 8d.

163. How many yards of Stuff, $\frac{1}{2}$ a yard wide, are equal to $36\frac{1}{2}$ yards, of $\frac{3}{4}$ wide?   Answer, $54\frac{3}{4}$ yards.

164. Suppose $6\frac{1}{2}$ yards cost 18s., what cost $9\frac{1}{4}$ yards?

Answer, £1. 5s. $7\frac{1}{4}$d.$\frac{7}{13}$

165. If $3\frac{1}{4}$ ounces of Silver cost $19\frac{3}{4}$ shillings, what cost $29\frac{1}{2}$ ounces?   Answer, £8. 9s. $4\frac{3}{4}$d. $\frac{5}{9}$

166. Bought $\frac{9}{16}$ of a Cwt. for £$14\frac{1}{2}$, what cost $7\frac{1}{2}$ Cwt.?

Answer, £118. 6s. 8d.

167. If $\frac{1}{2}$ of a gallon of Wine cost £$\frac{1}{8}$, what will $\frac{1}{2}$ of a tun cost?

Answer, £140.

168. If 9 men can perform a piece of work in $15\frac{1}{2}$ days, in what time would 23 men do the same?   Answer, $6\frac{1}{23}$ days.

169. If $8\frac{1}{4}$ lb. of Tobacco are sold for $22\frac{1}{4}$ shillings, what cost 1 lb.?

Answer, 2s. $8\frac{3}{4}$d. $\frac{2}{33}$

170. Paid £$3\frac{1}{4}$ for $2\frac{1}{4}$ yards, what would $17\frac{1}{2}$ yards cost?

Answer, £27. 13s. $6\frac{3}{4}$d. $\frac{2}{9}$

171. A lends B £$35\frac{1}{4}$ for $6\frac{1}{2}$ months; how long ought B to lend A £$12\frac{1}{4}$, to requite the kindness?   Answer, 18 mos. 22 days $\frac{2}{4\frac{1}{6}}$

172. If $15\frac{1}{4}$ gallons of Beer cost £$1\frac{3}{4}$, what is that per pint?

Answer, $2\frac{1}{4}$d. $\frac{49}{183}$

173. What length of Cloth $1\frac{1}{4}$ yard wide, is equivalent to $20\frac{1}{4}$ yards of $3\frac{1}{4}$ wide?   Answer, 53 yds. 1 qr. $\frac{1}{2}$

174. If 1 yard of Broad Cloth cost $15\frac{5}{8}$ shillings, what will 4 pieces, each containing $27\frac{1}{4}$ yards, cost?   Answer, £85. 10s. $11\frac{1}{4}$d.

175. If I have $7\frac{13}{14}$ Cwt. carried for 8s. $7\frac{1}{4}$d., what weight can I have carried for £$5\frac{5}{16}$?   Answer, 94 Cwt. 2 qr. 17lb.$\frac{1}{5}\frac{1}{7}\frac{9}{7}$

DECIMAL FRACTIONS.

Decimal Fractions are such as have 10, 100, 1000, &c. for their denominators; but are usually expressed by writing the numerator only, the understood denominator being always ten, or some power of ten, whence its name, (decem).

The denominator of every decimal fraction is an unit, with as many ciphers as there are figures in the decimal—thus, $\frac{5}{10}, \frac{75}{100}, \frac{327}{1000}, \frac{1875}{10000}$, &c. are written with a point before them, as ·5, ·75, ·327, ·1875; and are read, 5 tenths, 75 hundredths, 327 thousandths, 1875 ten thousandths, respectively.

Ciphers on the right-hand of decimals make no alteration in their value, for ·5, ·50, ·500, &c. are decimals, having the same value, being each equal to $\frac{1}{2}$; but if ciphers are placed on the left-hand, they decrease the value in a tenfold proportion—thus, ·5, ·05, ·005, are 5 tenth parts, 5 hundredth parts, and 5 thousandth parts.

A mixed number is composed of a whole number and a decimal, the figures *after* the point being decimals—

thus,   3 · 8   are read   3 and   8 tenths,
        72 · 94   „   72 and 94 hundredths,
        47 · 376   „   47 and 376 thousandths,
         1 · 003   „   1 and   3 thousandths.

ADDITION OF DECIMALS.

Example.

Find the sum of 8·57—·0056—295·702—1·37685—·217—15·8163—57·2.

Point.

```
  8.57
   .0056
295.702
  1.37685
   .217
 15.8163
 57.2
─────────
378.88775 Total.
```

Rule.

Place the numbers so that all the decimal points may fall directly under each other; and add as in whole numbers.

Exercises.

1. Find the sum of 27.6——39.213——7214.9——417.05—— and 5.0327.   Answer, 7703.7957.

2. Find the sum of 7.045——37.23——5.0047——.91864——396.243 and 4.19.   Answer, 450.63134.

3. Find the sum of 12.47——3.584——.75——29.047——1.8236 and 15.75.   Answer, 63.4246.

4. Find the sum of .05864——293.147——84.2361——5.07——.3274 and 15.943.   Answer, 398.77714.

5. Find the sum of 5.714——3.456——.5437——1.04——276.358 and 7.2.   Answer, 294.3117.

6. Find the sum of .005——7.296——4.318——.05947——28.216 and 7.92.   Answer, 47.81447.

7. Find the sum of 39.33——4.2056——.98735——4.6287——347.91 and 3.004.   Answer, 400.06565.

8. Find the sum of 2.53——.01825——17.5——.00375——199.25——144——1431.7256 and 58.729.   Answer, 1853.7566.

SUBTRACTION OF DECIMALS.

Example.
81. 4763
9. 580936

71. 895364 Difference.

RULE.

Place the numbers so that the decimal points may be under each other; and subtract, as in whole numbers.

EXERCISES.

9. From   127. 62 take   13. 725.   Answer, 113. 895.

10. From 6213. 725 take   162. 25.   Answer, 6051. 475.

11. From 3760. 279 take 423. 0076.   Answer, 3337. 2714.

12. From   238. 73 take   24. 836.   Answer, 213. 894.

13. From   7. 10563 take   .9542.   Answer, 6. 15143.

14. From   103. take   . 00729.   Answer, 102. 99271.

15. From   407. 52 take 71. 30976.   Answer, 336. 21024.

16. From   2. 62875 take   . 007364.   Answer, 2. 621386.

MULTIPLICATION OF DECIMALS.

Example.
Multiplicand   27. 3296
Multiplier ..   45. 839

2459664
819888
2186368
1366480
1093184

1252·7615344 Product.

RULE.

Multiply the decimals as if they were whole numbers; and from the product point off so many places for decimals as there are decimal places both in the multiplicand and multiplier.

Four decimals (*viz.* 3296) in the multiplicand, and three places (*viz.* 839) in the multiplier; consequently 7 decimal places (7615344) are pointed off the product.

Example.
Multiplicand   . 35786
Multiplier ..   . 0467

250502
214716
143144

. 016712062 Product.

If the decimal places in the multiplicand & multiplier exceed those in the product, supply the defect by adding ciphers to the left-hand.

17. Multiply 27.548 by 42.35.   Answer, 1166.65780.
18. Multiply 79.347 by 2.135.   Answer, 169.405845.
19. Multiply 6.3478 by .8204.   Answer, 5.20773512.
20. Multiply .02534 by .03256.   Answer, .0008250704.
21. Multiply 2.7964 by   .385.   Answer, 1.076614.
22. Multiply .385746 by .00464.   Answer, .00178986144.
23. Multiply 51807.3 by .29476.   Answer, 15270.719748.

DIVISION OF DECIMALS.

Example.

Divisor. Dividend. Quotient.
4.35) 27.462837 (6.3132

           1362

            578

           1433

           1287

           .417 Remainder.

RULE.

Divide as in whole numbers, and point off from the right-hand of the quotient as many figures for decimals as the decimal places in the dividend exceed those in the divisor.

Here are six decimal places in the dividend (*viz.* 462837), and two places in the divisor (*viz.* 35), the excess of the dividend above the divisor being 4 places: consequently 4 places (3132) are pointed off from the quotient.

Example.

Divisor. Dividend. Quotient.
1117) 9.3250496 (.0083482

           3890

           5394

           9269

           3336

           1102 Remainder.

Here the dividend contains 7 decimal places, but the divisor consists of whole numbers; therefore the places in the dividend exceed those in the divisor by the 7: but as the quotient contains only 5 figures, the defect is supplied by prefixing 2 ciphers.

Proof of the foregoing.

Multiplicand.. .0083482
Multiplier....   1117

———————

1419194
918302
     1102 Remainder.

———————

9.3250496 Product.

EXERCISES.

24. Divide   67.289544 by 1.764.   Answer, 38.146.
25. Divide 273.50856 by   3.9.   Answer, 70.1304.
26. Divide   728.6429 by 5.18.   Answer, 140.66, &c.
27. Divide 234.76958 by   7.   Answer, 33.53851, &c.
28. Divide   8.572046 by   43.   Answer, .199349, &c.
29. Divide .028734619 by .057.   Answer, .504116, &c.
30. Divide   .91730846 by .162.   Answer, 5.66239, &c.

Example.

Divide 179 by 369.5.

Divisor.   Dividend.   Quotient.
369.5) 179.000000 (.48443

———————

31200

———————

16400

———————

16200

———————

14200

———————

3115 Remainder.

———————

Proof.

Multiplicand .48443
Multiplier..   369.5

———————

242215
435987
290658
145329
      3115 Remainder

———————

Product 179.000000

When the dividend requires it, annex 6 ciphers at first.— Should the quotient be wanted beyond, place the ciphers as you proceed, and point off the quotient accordingly.

31. Divide 83 by 4.79.   Answer, 17. 3277, &c.
32. Divide 175 by 2.075.   Answer, 84. 3373, &c.
33. Divide 13 by .7296.   Answer, 17. 8179, &c.
34. Divide 5 by 47.   Answer, . 1063829, &c.

REDUCTION OF DECIMALS.

CASE 1st. To reduce a Vulgar Fraction to a Decimal.

Example.

Reduce ⅞ to a decimal of equal value.

$$8)\,7.000$$
$$.875$$

Answer, . 875.*

Another Example.

Reduce ¹⁹⁄₃₇ to a decimal of equal value.

$$37)\,19.000000\,(.513513,\ \&c.$$
.50
.130
.190
.50
.130
.19

RULE.

Annex as many ciphers to the numerator as may be required: then divide by the denominator, and point off so many places in the quotient for decimals as there were ciphers annexed.

NOTE. Decimals extended to six places, are usually sufficient for most calculations.

The continual repetition of the same figure or figures (as in this quotient) is called a recurring or circulating decimal.

---

* The truth of this Rule will appear, by expressing the decimal with its denominator, and reducing it to lowest terms, (Vul. Frac. Case 2d.) thus,— $\frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$.

EXERCISES.

35. Reduce $\frac{1}{2}$ to a decimal of equal value.   Answer,   . 5

36. Reduce $\frac{1}{4}$ to a decimal of equal value.   Answer,   . 25

37. Reduce $\frac{3}{4}$ to a decimal of equal value.   Answer,   . 75

38. Reduce $\frac{5}{8}$ to a decimal of equal value.   Answer, . 625

39. Reduce $\frac{3}{13}$ to a decimal of equal value.

Answer, . 230769, &c.

40. Reduce $\frac{11}{14}$ to a decimal of equal value.

Answer, . 785714, &c.

41. Reduce $\frac{3}{25}$ to a decimal of equal value.   Answer, . 12.

42. Reduce $\frac{1}{32}$ to a decimal of equal value.   Answer, . 03125.

43. Reduce $\frac{16}{27}$ to a decimal of equal value.

Answer, . 592592, &c.

44. Reduce $\frac{17}{243}$ to a decimal of equal value.

Answer, . 069958, &c.

45. Reduce $\frac{143}{107}$ to a decimal of equal value.

Answer, . 787439, &c.

46. Reduce $\frac{295}{3172}$ to a decimal of equal value.

Answer, . 0930012, &c.

47. Reduce $\frac{142}{125}$ to a decimal of equal value.

Answer, . 0109953, &c.

CASE 2d.   To find the value of a given decimal, in terms of the inferior denomination.

Example.

What is the value of .1275 of a pound ?

```
   . 1275
      20
  ————————
  2,5500
      12
  ————————
  6,60
      4
  ————————
  2,4
```

Answer, 2s. 6½d. .4

RULE.

Multiply the decimal by the number of parts in the next lower denomination, and point off, as in Multiplication ; and so proceed through each denomination.

Multiply by 20, 12, and 4, the same as in Reduction descending, bringing great names into small.

Note.—All ciphers on the right hand may be rejected.

F

Example.

What is the equivalent of
.918653 of a hogshead of Wine?

      .918653
         63
      ———
    2755959
    5511918
    ———
   57.875139
        4
    ———
    3.500556
       2
    ———
    1.001112
    ═══

Answer, 57 galls. 3 qts. 1 pt.

Multiply by 63 (galls.=1 hhd.)
and by 4 (qts. =1 gal.)
and by 2 (pts. =1 qt.)

EXERCISES.

48. What is the equivalent of . 625 of a shilling ?   Answer, 7½d.

49. What is the equivalent of . 225 of a pound?   Answer, 4s. 6d.

50. What is the equivalent of . 5728 of a pound troy ?
                   Answer, 6 oz. 17 dwt. 11 gr. . 328

51. What is the equivalent of . 575 of Cwt.
                  Answer, 2 qrs. 8 lb. 6 oz. 6 drs. . 4

52. What is the equivalent of . 375 of a yard of Cloth ?
                         Answer, 1 qr. 2 nls.

53. What is the equivalent of . 0628 of a barrel of Beer ?
                  Answer, 2 galls. 1 qt. . 0432

54. What is the equivalent of . 7575 of a pound? Ans. 15s. 1¾d. .2

55. What is the equivalent of . 375 of a guinea?   Ans. 7s. 10½d.

56. What is the equivalent of . 04535 of a mile ?
          Answer, 14 poles, 2 yds. 2 ft. 5 in. 1 b. c. . 128

57. What is the equivalent of . 42857 of a month ?
      Answer, 1 wk. 4 days, 23 hrs. 59 min. 56 sec. . 544

58. What is the equivalent of . 2765 of a year ?
        Answer, 100 days, 23 hrs. 47 min. 56 sec. ⅖

Case 3d. To reduce numbers of different names to their equivalent decimal values.

RULE.

Place down the lowest name mentioned. Divide this by as many as make one of the next higher denomination, annexing ciphers to the dividend as you require.

Before this quotient place the next higher name, and divide by as many as make one of the next higher denomination.

And so on throughout, and the last quotient is the decimal required.

Example.

Reduce 9s. 2¾d. to the decimal of a pound.

    4) 3.00
    ─────
   12) 2.75
    ─────
   20) 9.22916, &c.
    ─────
     .461458, &c.

Answer, .461458 of a £.

Here, the lowest name is 3 fgs. divided by 4, because 4 fgs.=1d., with two ciphers annexed to the dividend.

The next higher name is 2d., which is placed before .75. 2.75 is divided by 12, (12d.=1 sg.)

To this quotient 9 is placed, and 9.22916 is lastly divided by 20, (20 sgs. = 1 £). So that 9s. 2¾d.=461458 millionths of a pound.

Another Example.

Reduce 2 qrs. 7 lb. 8 oz. to the decimal of a Cwt.

$$16\begin{cases}4) & \dfrac{8.}{} \\ 4) & \dfrac{2.}{7.5} \end{cases}$$

$$28\begin{cases}4) & \dfrac{}{1.875} \\ 7) & \dfrac{}{2.267857} \end{cases}$$

    4) ─────
      .566964, &c.

Answer, .566964, &c. of a Cwt.

The lowest name is 8 oz. divided by 16, (16 oz. = 1 lb.)

The next higher name is 7 lb., which is placed before .5. 7.5 is divided by 28, (28 lb.=1 qr.), and to this quotient the 2 qrs. are placed. Lastly, 2.267857 is divided by 4, (4 qrs.=1 Cwt.) So that 2 qrs. 7 lbs. 8 oz.=566964 millionths of a Cwt.

This and the last Case prove each other.

Proof of the preceding Example by the last Case.

What is the equivalent of .566964, &c. of a Cwt.

.566964, &c.
4

———————

2.267857 (with Remainder.)
28

———————

2142860 (with Remainder.)
535714

———————

7.500000
16

———

8.0

Answer, 2 qrs. 7 lb. 8 oz.

EXERCISES.

59. Reduce 9½d. to the decimal of a shilling.

Answer, .791666, &c.

60. Reduce 14s. 6d. to the decimal of a £.   Answer, .725

61. Reduce 17s. 5¼d. to the decimal of a £.   Answer, .871875

62. Reduce 5 oz. 12 dwts. 7 grs. to the decimal of a pound troy.

Answer, .4678819, &c.

63. Reduce 1 rood, 30 poles, to the decimal of an acre.

Answer, .4375

64. Reduce 3 qrs. 18 lb. to the decimal of a Cwt.

Answer, .910714

65. Reduce 3 bush. 1 pk. to the decimal of a quarter.

Answer. .40625

66. Reduce 1 qr. 1 nl. to the decimal of a yard.   Answer, .3125

67. Reduce 3 weeks, 6 days to the decimal of a month.

Answer, .964285, &c.

68. Reduce 7s. 10½d. to the decimal of a guinea.   Answer, .375

69. Reduce 18s. 11¾d. to the decimal of a pound.

Answer, .948958, &c.

70. Reduce 3 qrs. 17 lbs. 9 oz. 15 dr. to the decimal of a Cwt.

Answer, .907331, &c.

RULE OF THREE IN DECIMALS.

~~~~~~~~~~~~~~~~~~~~~

RULE. If the given numbers require it, reduce them to decimals by the last Case.

State the question, as in the common Rule of Three :—multiply the second and third terms together, and divide by the first:—the quotient is the answer, in the same name as that to which the last term was reduced.

Example.

If 5 Cwt. 2 qrs. 16 lb. of Sugar cost £25. 18s. 9½d., what cost 4½ lb. ?

lb.	lb.	fgs.	The three
16	4.5	2.	terms are first
28 $\{$ 4————	28 $\{$ 4———	4———	reduced to
4.	1.125	9.5	Decimals
7———	7———	12———	by the last
2.571428	.160714	18.79166	Case.
4 ———	4 ———	20———	
Cwt. 5. 64285	Cwt. .040178	£25.93958	

			Then stated,
Cwt.	Cwt.	£	as directed in
5.64285 :	.040178 : :	25.93958	the Rule of
		.040178	Three.

```
                          20751664
                          44097286
                         10375832
                       ——————————      £
        5.64285) 1.04220044524 (.184693
                  —————————
                    4779154
                  —————————
                    2648744
                  —————————
                   .3916045
                  —————————
                    .5303352
                  —————————
                    .2247874
                  —————————
                     .555019
                  —————————
```

$$
\begin{array}{r}
\pounds \\
.184693 \\
20 \\
\hline
3.693860 \\
12 \\
\hline
8.32632 \\
4 \\
\hline
1.30528 \\
\hline
\end{array}
$$

The Quotient
brought to its
eqivalent
value.

Answer, 3s. 8½d.+.30528.

EXERCISES.

71. How much Pepper, at 1s. 4½d. per lb., can I buy for £12. 15s. 9d.?
Answer, 1 Cwt. 2 qrs. 18 lb.

72. If £250. 10s. gain £12. 10s. 6d. in 12 months, what principal will gain the same in 7½ months?   Answer, £400. 16s.

73. How much Stuff, ⅜ of a yard broad, will line 7¾ yards of Cloth, ⅞ of a yard broad?   Answer, 9 yards +.0416.

74. At £2. 19s. per ounce, what is the value of 19 oz. 3 dwts. 5 grs. ?
Answer, £56. 10s. 5½d.+.2981.

75. At 12s. 4d. per ell English, required the value of 49¼ yards.
Answer, £24. 5s. 11d.+.1984.

76. If silver be worth 4s. 7d. per ounce, what is the value of 2 lb. 5 oz. 17 dwts. 5 grs. ?   Answer, £6. 16s. 10½d.+.2867.

77. If 4 yds. 3 qrs. of Cloth cost £1. 5s. 7½d., how many yards can be purchased for £20. 13s. 9d. ?
Answer, 76 yds. 2 qrs. 3 nls.+.1216.

78. How many Cwt. of Sugar can be purchased for £36.75, at 9½d. per lb. ?

79. If 3½ yards of Cloth were sold for 12s. 1½d., what would be the cost of 7 pieces, each 25¾ yards ?

INVOLUTION.

Involution, or the raising of powers, is the method of finding the Square, Cube, &c. of any given number, and is performed by repeated multiplication.

Any number multiplied into itself produces the square, or second power; and that product, multiplied by the given number, produces the cube, or third power; and so on to any power whatever.

The number given to be involved, is called the Root, or first power.

The index, or exponent of a power, is the number which denotes the power: the square of 9 is often expressed 9^2; the cube of 8 as 8^3; the 4th power, or biquadrate of 23 as 23^4, &c.

Example.

What is the 4th power of 45 ?

```
    45  Root.
    45
   ----
   225
   180
   ----
  2025  Square.
    45
   ----
 10125
  8100
 -----
 91125  Cube.
    45
 -----
455625
364500
-------
4100625  Biquadrate.
```

RULE.

Multiply the given number by itself for the square, and the square by the given number for the cube; and so on to any power required.

Note. — Or, for the fourth power, multiply the square by the square. $2025 \times 2025 = 4100625$, the fourth power.

EXERCISES.

1. What is the square of 27 ?
2. What is the cube of 139 ?
3. What is the 4th power of 78 ?
4. What is the 5th power of 35 ?

Answer, 729.
Answer, 2685619.
Answer, 37015056.
Answer, 52521875.

When Vulgar Fractions are to be involved.

Example.

What is the 3d power of $\frac{3}{4}$?

Numerator.	Denominator.
3 Root.	4 Root.
3	4
——	——
9 Square.	16 Square.
3	4
——	——
27 Cube.	64 Cube.
=	=

Answer, $\frac{27}{64}$

RULE.

Raise both the numerator and denominator separately to the required power.

EXERCISES.

5. What is the square of $\frac{4}{5}$?
6. What is the cube of $\frac{3}{7}$?
7. What is the 4th power of $\frac{2}{3}$?
8. Required the square of $\frac{3}{4}$.

Answer, $\frac{15}{24}$
Answer, $\frac{343}{512}$
Answer, $\frac{16}{615}$
Answer, $\frac{841}{1200}$

When Decimal Fractions are to be involved.

Example.

What is the 4th power of . 037 ?

```
          . 037 Root.
          . 037
          ——————
           259
           111
     ——————————————
       . 001369 Square.
       . 001369
     ——————————————
         12321
          8214
         17797
  ——————————————————————
  . 000001874161 Fourth power.
```

RULE.

Raise them like whole numbers, pointing off each product the same as Multiplication of Decimals, page 93.

9. What is the square of 1.52 ?   Answer, 2.3104.
10. What is the cube of .54 ?   Answer, .157464.
11. What is the 4th power of .013 ?   Answer, .000000028561.
12. What is the square of 82.17 ?   Answer, 6751.9089.

EVOLUTION.

Evolution, or the Extraction of Roots, is the method of finding the Square Root, Cube Root, &c. of a given number; being the reverse of Involution.

Roots are generally indicated by the radical sign $\sqrt{}$ before the numbers: thus, $\sqrt{25}$, implies that the square root of 25 is to be extracted; $\sqrt[3]{27}$, the cube root 27, &c.; but sometimes they are expressed by a fraction placed over the number thus, $\overline{25}|^{\frac{1}{2}}$; $\overline{27}|^{\frac{1}{3}}$.

When no exact root of any number can be found, it is called a Surd Root.

To Extract the Square Root of a Number.

RULE.

Place a point, or some other mark, over the right-hand or units figure, and every other figure to the left.

Find the nearest root in the first period, and place it on the right-hand, as a quotient, and the square under the said period.

Subtract the square, and bring down the next two figures to the remainder for a dividend.

Double the quotient for a divisor, and seek how often it is contained in the dividend, omitting the units place, and

put the figure supposed, in the quotient, and also in the units place of the divisor.

Multiply the divisor by the figure last put in the quotient, and subtract the product from the dividend.

Bring down the next two figures, find a divisor as before, repeating the foregoing directions to the end.

Example.

What is the square root of 125316?

```
          . . .
    125316 (354
    9
    ——
 64) 353
     325
     ———
704) 2816
     2816
     ————
     . . . .
```

Answer, 354 square root.

Point over the last figure, 6, and every other to the left. Then the nearest root in the first period, 12, is 3. Put 3 in the quotient; and 3 squared=9, which put under 12. Subtract, and 3 remain, to which bring down the next two figures, 53: consequently 353 is a new dividend. Double the quotient, 3, which is 6, and place 6 for a divisor, leaving the units place. 6 in 35, 5, (with a remainder.) Put 5 in the quotient, and also in the units place of the divisor. Multiply the divisor, 65, by 5, which is 325. Subtract 325 from 353, and 28 remain. Bring down the next two figures, 16; consequently 2816 is a new dividend. Double the quotient, 35, which is 70. Put 70 for a divisor, leaving the units place; 70 in 281 are 4. Put 4 in the quotient, and also in the units place of the divisor. Multiply the divisor, 704, by 4, which is 2816, the same as the dividend.

Example.

What is the square root of 5307705316 ?

```
              . . . . .
    5307705316 (72854
    49
    ——
  142) 407
       284
       ———
 1448) 12370
       11584
       —————
14565) 78653
       72825
       —————
145704) 582816
        582816
        ——————
        . . . . . .
```

Point over 6, and every other figure to the left. The nearest root in the first period, 53, is 7. Put 7 in the quotient: and 7 squared=49. 49 from 53, 4 remain, to which bring down the next two figures, 07; consequently, 407 is a new dividend. Double the quotient, 7, which is 14. Put 14 for a divisor, leaving the units place. 14 in 40, twice (with a remainder). Put 2 in the quotient, and also in the units place of the divisor. Multiply the divisor, 142, by 2. Subtract 284 from 407, and 123 remain. Bring down the next two figures, 70. Consequently, 12370 is a new dividend. Double the quotient 72=144. Put 144 for a divisor; leaving the units place; 144 in 12370 are 8. Put 8 in the quotient and also in the units place of the divisor. Multiply the divisor, 1448, by 8, which is 11584. Subtract 11584 from 12370, and 786 remain. And so by repeating the rule, we proceed to the end.

Proof of the last Example by Involution.

$$
\begin{array}{r}
72854 \\
72854 \text{ Root.} \\
\hline
291416 \\
364270 \\
582832 \\
145708 \\
509978 \\
\hline
5307705316 \text{ Square.}
\end{array}
$$

EXERCISES.

1. What is the square root of 625 ?   Answer, 25.
2. What is the square root of 59049 ?   Answer, 243.
3. What is the square root of 289444 ?   Answer, 538.
4. What is the square root of 2187441 ?   Answer, 1479.
5. What is the square root of 6285049 ?   Answer, 2507.
6. What is the square root of 5300568025 ?   Answer, 72805.

When the given number has no exact square root, or is a Surd.

Example.

What is the square root of 34 ?

$$
\begin{array}{r}
34\,(5.830951, \&c. \\
25 \\
\hline
108)\,900 \\
864 \\
\hline
1163)\,3600 \\
3489 \\
\hline
116609)\,1110000 \\
1049481 \\
\hline
1166185)\,6051900 \\
5830925 \\
\hline
11661901)\,22097500 \\
11661901 \\
\hline
10435599
\end{array}
$$

Answer, 5.830951, &c.

Add two ciphers to each remainder, and point off the quotient, where you begin the addition of these ciphers, for decimals.

The quotient extended to 6 decimal places is sufficient for practical purposes.

F 6

7. What is the square root of 18 ?   Answer, 4.24264, &c.
8. What is the square root of 127 ?   Answer, 11.2694, &c.
9. What is the square root of 3857 ?   Answer, 62.1047, &c.
10. What is the square root of 84276?   Answer, 290.3032, &c.

When it is required to extract the square root of a number with decimals annexed.

Example.
What is the square root of
378.52917 ?

```
. .   . .
378.52917 (19.4558, &c.
```

29) 278

384) 1752

3885) 21691

38905) 226670

389108) 3214500

.101636

Answer, 19.4558, &c.

Place a point, or some other mark, over the units figure in the whole number, and then point every other figure to the right and left of it, as before.

The work may be shortened by using the contracted way of performing Division. (Vide page 16).

11. What is the square root of 19.0969 ?   Answer, 4.37.
12. What is the square root of 571.0463 ?   Answer, 23.8965, &c.
13. What is the square root of 15.376 ?   Answer, 3.92122, &c.
14. What is the square root of 28973.4 ?   Answer, 170.215, &c.
15. Required the square root of 8 ?
16. Extract the square root of 53202436 ?
17. What is the square root of 703.28761 ?
18. Required the square root of 15241578750190521 ?

Application of the Rule of Three;

SIMPLE PROPORTION.

SIMPLE INTEREST.

Principal is the money lent.

Interest is the money paid for the use of the Principal.

Rate per Cent. is a certain sum agreed on between the Lender and the Borrower, for the use of every £100 Principal.

Amount is the Principal and Interest added together.

Time relates to the years, months, and days for which the Principal is lent.

Example.

What is the yearly interest of £875. 15s. at £3½ per Cent. ?

```
  £       £    s.        £
 100  : 875  15  : :   3½
               3
            ─────────
          2627   5
      ½    437  17   6
          ─────────
         30,65   2   6
             20
          ─────────
         13,02
             12
          ─────────
          0,30
              4
          ─────────
          1,20
          ─────────
```

By the Directions given in the Rule of Three, the Answer must be Interest, consequently £3½ is put for the 3d term. The Answer must be greater; therefore £875. 15s. is put for the 2d term, &c.

Multiply by 3½. In dividing by 100, retain the units and tens place (65), and cut off the others (30) for the pounds. From the figures retained, the shillings, &c. are found.

Answer, £30. 13s. 0½d.

Example.

What is the amount of £1750. 10s. 6d. for 3½ years, at £4 per Cent. per annum?

£		£	s.	d.		£
100	:	1750	10	6	::	4
				4		

70,02   2   0
       20

0,42
  12

5,04

First, the Interest for one year is found, as before shewn;

	£	s.	d.	
½	70	0	5	Interest for 1 year.
			3	
	210	1	3	Interest for 3 years.
	35	0	2½	Interest for ½ year.
	1750	10	6	Principal.

Answer, 1995 11 11½ Amount for 3½ years.

And then multiplied by the number of years.

The principal is added to give the Amount.

Example.

Required the interest of £793. 18s. for 2 yrs. 5 mos. 18 days, at £3¾ per Cent.

£		£	s.		£
100	:	793	18	::	3¾
			3		

	2381	14	
⅓..½	396	19	
¼..½	198	9	6

29,77   2   6
       20

15,42
   12

5,10

	£	s.	d.		
	29	15	5	Interest for 1 year.	
	29	15	5	Do.	Do.
4 mos.=⅓	9	18	5½	Do.	4 mos.
1 mo. =¼	2	9	7½	Do.	1 mo.
15 days=½	1	4	9½	Do.	15 days.
3 days=⅕	0	4	11½	Do.	3 days.

Take parts for the months and days, as in Practice.

Answer, £73 8 7¾   Do.   2 years, 5 mos. 18 days.

EXERCISES.

1. Required the interest of £520. 10s. for a year at 4 per Cent.
                                    Answer, £20. 16s. 4⅘d.

2. What is the interest of £453. 5s. for 3 years, at £5 per Cent.?
                                    Answer, £67. 19s. 9d.

3. What is the amount of £278. 18s. 6d. for 2½ years, at £3½ per
Cent. per annum?   Answer, £301. 11s. 9d.

4. Required the interest of 500 guineas for 9 years, at £4½ per
Cent.   Answer, £200. 16s. 3d.

5. What is the interest of £1784. 4s. 8d. for 37 weeks, at £5 per
Cent. per annum?*   Answer, £63. 9s. 6¼d.

6. Required the interest of £2745. 11s. 9d. for 125 days, at £3½
per Cent.*   Answer, £32. 18s. 2d.

7. Required the amount of £494. 12s. 10d. for 41 weeks, at £4½
per Cent.   Answer, £512. 3s. 10d.

8. What is the interest of £842. 13s. 10d. for 3 yrs. 8 mos. 12 days,
at £4 per Cent.?   Answer, £124. 14s. 3¼d.

9. What is the interest of £532. 15s. from the 4th of March to
the 23d of September, both days inclusive, at £3 per Cent.?
                                    Answer, £8. 18s. 7¾d.

10. Required the amount of £1970. 4s. for 5 yrs. 7 mos. 24 days,
at £3½ per. Cent.   Answer, £2359. 16s.

---

* To find the Interest for weeks, state—
As 52 weeks : given weeks :: interest of 1 year, to the interest required.
    To find the Interest for days—
As 365 days : given days :: interest for 1 year, to the interest required.

COMMISSION and BROKERAGE.

Commission is an allowance of a certain rate per Cent. to a factor or agent, for buying or selling goods for his employer.

Brokerage differs from Commission chiefly in the rate being less than £1 per Cent.

Example.

What is the commission on £978. 15s. 8d., at £2¼ per Cent. ?

```
£           £   s.  d.        £
100    :   978  15  8    : :   2¼
           978  15  8
      ¼    244  13  11
         ──────────────
           22,02   5   3
                 20
              ──────
               0,45
                 12
              ──────
               5,43
                  4
              ──────
               1,72
              ══════

      Or by Practice.
              £   s.   d.
             978  15   8
  £        ──────────────
2 = 1/50     19  11   6
¼ = ⅛         2   8  11¼
         ──────────────
Answer, £22   0   5¼
         ══════════════
```

This is similar to finding the interest for one year.

Exercises.

1. Required the commission on £427. 18s., at £2 per Cent.

Answer, £8. 11s. 1¾d.

2. Required the commission on £615. 14s. 3d., at £3½ per Cent.

Answer, £21. 10s. 11¾d.

3. Required the commission on £1430. 3s. 6d., at £2¼ per Cent.

Answer, £35. 15s. 1d.

4. What is the commission on £590. 19s., at £1¼ per Cent.?

Answer, £7. 7s. 8¼d.

5. If a factor be allowed £2¼ per Cent., what does his commission amount to on 1000 guineas?   Answer, £27. 11s. 3d.

6. A merchant in London allows his factor in America £1¾ per Cent. commission; what may he demand on £2078. 15s.?

Answer, £38. 19s. 6¼d.

Example.

Required the brokerage on £527. 14s. 1d., at ⅛ per Cent.

£		£	s.	d.		s.	d.
100	:	527	14	1	::	12	6

		£	s.	d.
10s.=½		263	17	0½
2s. 6d.=⅛		65	19	3
		3,29	16	3¼
		20		
		5,96		
		12		
		11,55		
		4		
		2,22		

Take parts of the proposed rate, and divide by 100 for the brokerage.

Answer, £3. 5s. 11½d.

7. Required the brokerage on £675. 14s. at ¾ per Cent.

Answer, £5. 1s. 4¼d.

8. Required the brokerage on £1240. 10s. at ⅜ per Cent.

Answer, £4. 13s. 0¼d.

9. What is the brokerage on £1053. 1s. 8d. at ½ per Cent.?

Answer, £5. 5s. 3¼d.

10. If a broker be allowed 5s. 6d. per Cent., what will the brokerage amount to on £829. 17s. 6d.?   Answer, £2. 5s. 7½d.

THE STOCKS,

OR

PUBLIC FUNDS.

Stock was a term originally used to signify the capital of Public Companies, as that of the Bank of England, the East India Company, South Sea Company, &c.

It is now chiefly applied to the Public Funds, or Debts of Government.

There are various kinds of Stock, as the 3 per Cent. Consolidated Annuities; New £3. 10s. per Cent. Consolidated Annuities; 3 per Cent. Reduced Annuities; Bank Stock; India Stock, &c. &c.

These Stocks are transferable at pleasure, even in small sums, and the interest is paid regularly, every half-year.

The price per £100 of the Stocks varies according as political and other circumstances influence the market.

The practice of funding began A. D. 1690, being money borrowed by Government at a certain rate per Cent. to supply the exigencies of the State; the total amount of the Government Securities, at this time, being about 800 millions.

To find the cost of any quantity of Stock.

Example.

What is the cost of £753. 15s. in the 3 per Cent. Consols, at £78¼ per Cent.—Brokerage ⅛ per Cent.?

```
  £              £    s.              £   s.  d.
 100    :      753  15      : :      78   2   6
                   11                      2   6  Brokerage.
              ─────────             ─────────────
               8291   5              78   5   0
                      7
              ─────────
              58038  15
                753  15
 5s...¼         188   8   9          ⅛th per Cent., or 2s. 6d.
              ─────────              on every £100, is paid to
              589,80  18  9          the Broker for buying or
                      20             selling the Stock.
              ─────────
               16,18
                  12
              ─────────
               2,25
                  4
              ─────────
               1,00
              ═════════
```

Answer, £589. 16s. 2¼d.

Or by Practice.

```
                         £    s.
                        753   15
                     ─────────────
 50=½                   376  17   6
 25=¼                   188   8   9
  2=¹⁄₅₀                 15   1   6
  1=½                     7  10   9
 5s.=¼                    1  17   8½
                     ─────────────
                     £589  16   2¼
                     ═════════════
```

EXERCISES.

1. What is the cost of £520. 15s. in the New 3½ per Cents., at £89¼ per Cent.? Answer, £465. 8s. 4¾d.

2. What is the cost of £987. 10s. in the 3 per Cents. Reduced, at £93¼ per Cent. Answer, £920. 16s. 10½d.

3. Required the cost of £1075 in the 3 per Cent. Consols, at £72½ per Cent.?   Answer, £782. 1s. 3d.

4. Required the cost of £500 Reduced 3½ per Cents., at £85½ per Cent.?   Answer, £427. 10s.

To find what quantity of Stock may be bought for, or sold, to produce a given sum.

Example.

How much Stock, at £78½ per Ct., can be bought for £589. 16s. 2½d.?

```
      78  2  6   :     589 16  2½   : :   100
          2  6*      =566217 fgs.
      ─────────              190
      78  5  0         ─────────       £
     =75120 fgs.   7512) 5662170,0 (753

                        40377                This Example
                                             proves the pre-
                        28170                ceding.

                        5634
                          20
                    ─────────
                7512) 112680 (15s.

                        37560
                        ─────
                        ....
```

Answer, £753. 15s.

EXERCISES.

5. What quantity of Stock in the New 3½ per Cents., at 85½ per Cent., can be bought for £700?   Answer, £815. 2s. 9d.

6. How much Stock in the 3 per Cent. Consols, at £72½ per Cent., will £478. 18s. 6d. purchase?   Answer, £662. 17s. 5½d.

7. What quantity of Stock, in the 3 per Cents. Reduced, at £80½ per Cent., must be sold to produce £725. 14s.?

Answer, £902. 17s. 10d.

8. How much Stock, at £62½ per Cent., must be sold to produce £317. 18s.?   Answer, £511. 14s. 2½d.

9. What is the difference on £1400, 3 per Cents. bought at £93½, and sold for £87½?   Answer, £89. 5s.

---

* When Stock is bought, *add* the Brokerage; if sold, *subtract.*

FELLOWSHIP,

IS EITHER

SINGLE OR DOUBLE.

SINGLE FELLOWSHIP

Is when different stocks are employed in trading, without reference to time.

By this Rule are adjusted the gains or losses of partners in company ; or the effects of bankrupts; or legacies, in case of a deficiency of assets or effects, &c.

Example.

Four partners, C, D, E, & F, engaged to trade in company : C's stock was £150, D's £320, E's £350, and F's £500: they gained £730.   Required how much of the gain each had to receive.

```
£150  C's Stock
 320  D's
 350  E's
 500  F's
 ─────      £       £
 132,0  :  73,0 : : 150  C's stock.
            150

      ⎧ 12) 10950
  132 ⎨     ─────
      ⎩  11) 912. 10
         ─────
```

C's share of the gain £82. 19   1+₁₁

RULE.   Add the several stocks together.   Then, as the whole stock is to the gain or loss, so is each person's stock respectively to his respective share of the gain or loss.

£   £   £
132,0   :   73,0   :: 320 D's stock.
                320
                ────
                1460
                219
                ────

        ⎧ 12) 23360
132 ⎨   ────────
        ⎩ 11) 1946. 13. 4
         ─────────────

D's share of the gain £176. 19. 4+$\frac{8}{11}$

£   £   £
132,0   :   73,0   :: 350 E's stock.
                350
                ────
                3650
                219
                ────

        ⎧ 12) 25550
132 ⎨   ────────
        ⎩ 11) 2129. 3. 4
         ─────────────

E's share of the gain £193. 11. 2+$\frac{8}{11}$

£   £   £
132,0   :   73,0   :: 500 F's stock.
                500
                ────

        ⎧ 12) 36500
132 ⎨   ────────
        ⎩ 11) 3041. 13. 4
         ─────────────

F's share of the gain £276. 10. 3+$\frac{7}{11}$

Proof.

£	s.	d.	
82	19	1$\frac{1}{11}$	C's sh.
176	19	4$\frac{8}{11}$	D's sh.
193	11	2$\frac{6}{11}$	E's sh.
276	10	3$\frac{7}{11}$	F's sh.
730	0	0	

EXERCISES.

1. A and B gained by trading £146; A put in £250, and B £370; what is their respective share of the gain?

Answer, £58. 17s. 5d. $\frac{1}{31}$ A's share.
£87. 2s. 6d. $\frac{30}{31}$ B's share.

2. Three men, A, B, and C, put in money together: A put in £23, B £32, and C £17; they gained £42. 15s.: required their respective shares of the gain.   Answer, £13. 13s. 1$\frac{1}{2}$d. A's share.
£19. 0s. 0d. B's share.
£10. 1s. 10$\frac{1}{2}$d. C's share.

3. Two merchants, L and M, made a stock of £210; of which L contributed £137, and M the rest: by trading they gained £60; what must each have of it?

<div align="right">

Answer, £39. 2s. 10¾d. ⁴⁄₇ L's share.
£20. 17s. 1¼d. ⁵⁄₇ M's share.

</div>

4. Divide £152 among three persons, so that their shares may be to each other as 3, 7, and 9.   Answer, £24, £56, and £72.

5. Two merchants trade together: A put into stock £500. 10s. 7d. and B 700 guineas; they gained £300. 15s.: what is each person's share thereof?   Answer, £121. 16s. 9d. A's share.

<div align="right">

£178. 18s. 2¾d. B's share.

</div>

6. Three merchant's freight a ship with 340 tuns of wine: D loaded 110 tuns, E 97, and F the rest. In a storm the seamen were obliged to throw 85 tuns overboard: how much must each sustain of the loss?

<div align="right">

Answer, D 27½, E 24¼, F 33¼.

</div>

7. Four men, E, F, G, and H, trade together; E put in £48, F £72, G £84, and H £25, and upon settling their accounts at the year's end, they found they had gained £175: what is each man's share of the gain?   Answer, £36. 13s.   7¼d. E's share.

<div align="right">

£55.   0s.   5d. F's share.
£64.   3s. 10d. G's share.
£19.   2s.   1d. H's share.

</div>

8. A person is indebted to K £238. 15s., to L £175. 19s., to M £360. 8s., and to N £53. 17s. 6d.; but his estate is worth only £500:—how must it be divided among them?

<div align="right">

Answer, £144.   0s.   0¾d. K's share.
£106.   2s.   6d. L's share.
£217.   7s.   6¼d. M's share.
£ 32.   9s. 11¾d. N's share.

</div>

9. A father left his estate, valued at £19090, among his three sons, in such a manner that for every £2 A gets, B shall have £3, and C £5:—how is the estate divided?

<div align="right">

Answer,   A has £3818.
B has £5727.
C has £9545.

</div>

DOUBLE FELLOWSHIP

Is when different stocks are employed for unequal times.

Example.

Three merchants, A, B, and C, engage in partnership. A puts in £547. 19s. 6d. for 7 months; B £475. 18s for 9 months; and C £1747. 14s. for 4 months: they trade, and gain £225. What is each person's share?

```
          £    s.  d.  Time.      £    s,   d.
  A's   547  19   6 × 7  = 3835  16   6
  B's   475  18   0 × 9  = 4283   2   0
  C's  1747  14   0 × 4  = 6990  16   0
                            ───────────────
                           15109  14   6
```

Then, 15109 14 6 : 225 :: 3835 16 6
(=604389 sixp.) (=9000 sixp.) 20

```
                    76716 sgs.
                       12
                    ───────
                    920598 pence.
                      9000      d.
            604389)8285382000 (13708
                    2241492
  12) 13708          4283250
  2,0) 1142,4        5252700
  Ans. £57. 2s. 4½d. A's share.   417588
                                       4
                          604389) 1670352 (½
                                  461574
```

Sixpences.   Sixpences.   £   s.
  604389 : 9000 ::   4283   2
Answer, £63. 15s. 7d. B's share.
Sixpences.   Sixpences.   £   s.
  604389 : 9000 ::   6990 16
Answer, £104. 2s. C's share.

Proof.
```
   57    2   4½+461574 A's share.
   63   15   7 +454308 B's share.
  104    2   0 +292896 C's share.
   ───────────
  £225    0   0
```

RULE.

Multiply each person's stock by the time of its continuance. Then, as the sum of all the products is to the gain or loss, so is each person's product to his share of the gain or loss.

EXERCISES.

10. D, E, and F trade in common. D puts in £100 for 6 months; E £160 for 8 months; and F £500 for 5 months: they gained £510. 15s.: required their proportionate shares of the gain.

Answer, £ 69. 19s. 3¼d.+ 54 D's share.
£149. 5s. 2¼d.+378 E's share.
£291. 10s. 5¾d.+ 6 F's share.

11. Three persons hire a pasture for £25 per annum. R. put in 14 horses for 5 months; S, 23 horses for 12 months; and T, 19 horses for 7 months. How much must each pay of the rent?

Answer, R pays £ 3. 13s. 0¾d.+147.
S pays £14. 8s. 1d. +388.
T pays £ 6. 18s. 9¾d.+423.

12. X put in a concern £2080 for 2 months; Y, £970 for 5 months; and Z, £400 for 15 months: they gain among them £650. What is the profit of each?

Answer, £180. 2s. 11d. +1060 X's profit.
£210. 0s. 6¼d.+ 875 Y's profit.
£259. 16s. 6¼d.+1067 Z's profit.

13. Four graziers take a piece of pasture land for a year, for which they pay 35 guineas. F put in 27 oxen for 11 months; G, 18 oxen for 7 months; H, 12 oxen for 9 months; and K, 33 oxen for 4 months. Required what each must contribute towards the rent?

Answer, F £16. 9s. 3d. +108.
G £ 6. 19s. 8d. +528.
H £ 5. 19s. 8¼d.+642.
K £ 7. 6s. 4d. + 48.

14. Three merchants enter into partnership, and trade as follows: A put in £150, and at the end of 7 months took out £50, and 5 months after that, he put in £170. B put in £205, and at the end of 5 months £110 more; but 4 months after, took out £150. C put in 300 guineas, and after 6 months took out £150; but 9 months after, put in £500. They continued trading for 18 months, when they found their gain to be £450. Required each person's share of it.

Answer. £115. 17s. 7¼d.+1740 A's share.
£137. 16s. 3¼d.+2380 B's share.
£196. 6s. 0¾d.+8190 C's share.

G

BARTER

Is the exchanging of one commodity for another, so that neither party may sustain loss by the exchange.

Example.

A and B barter goods. A has 479 yards of cloth at 3s. 5½d. per yard, which he exchanges with B for coffee at 2s. 9d. per lb.; what quantity of coffee must A receive?

Yards.
479 at 3s. 5½d. per yd.

		£	s.	d.	
3s. 4d. = ¼		79	16	8	
1d. = ₁/₄₀		1	19	11	
½d. = ⅛		0	19	11½	
		£82	16	6½	

The value of the Cloth is first found by Practice;

s.	d.		£	s.	d.			lb.
2	9	:	82	16	6½	:	:	1
12			20					
33d.			1656 sgs.					
2			12					
66			19878d.					
			2					

66) 39757 (602 lb.

..157

. 25

Then A must have a quantity of B's Coffee, proportionate to £82. 16s. 6½d., the value of his cloth.

The stating will appear, if put in the form of Simple Proportion, thus—how much Coffee at 2s. 9d. ℔ lb. can be had for £82.16s.6½d.?

Answer, 602 lb.=5 Cwt. 1 qr. 14 lb.

Exercises.

1. How many pounds of Cotton, at 17d. per lb., must be given for 47 reams of paper, at 15s. 9d. per ream?   Answer, 522₇/₁₇ lb.

2. How much Tobacco, at 2s. 8d. per lb., can be had for 4 Cwt. 1 qr. 18 lb. of Tea, at 5s. 6d. per lb.?   Answer, 9 Cwt. 10⅞ lb.

3. G has 5 pipes of Wine, at £30. 17s. per pipe, which he barters with H for Flannel, at 20½d. per yard; what quantity of Flannel must G receive?   Answer, 1805¹¹/₄₁ yards.

4. How many yards of Canvas, at 11d. per yard, must be given for 8 Cwt. 3 qr. 17 lb. of Raisins, at £4. 13s. 9d. per Cwt.?

Answer, 910$\frac{11}{11}$ yards.

5. What quantity of Pepper, at 14½d. per lb., must be given for 275 lb. of Ginger, at 3s. 7d. per lb.?   Answer. 7 Cwt. 1 qr. 17 lb.

6. D has 105 gallons of Brandy, at 15s. 10d. per gallon, for which he received of E £32. 16s., and 450 lb. of Sugar: what was the Sugar per lb.?   Answer, 2s. 2¾d. $\frac{3}{4}$

7. What quantity of Snuff, at 4s. 9d. per lb., must be given in barter for 3 Cwt. 1 qr. 17 lb. of Tobacco, at £7. 5s 8d. per Cwt.?

Answer, 104 lb.

~~~~~~

GAIN AND LOSS.

This Rule teaches to calculate what is gained or lost in the merchandise of goods, and instructs the trader to regulate the price of his goods, so that he may gain or lose so much per Cent.

Example.

If cloth is bought at 7s. 6d., and being damaged, sold at 6s. 2¼d., what is the loss per Cent.?

```
                   s.   d.
                   7    6   bought for.
                   6    2¼  sold for.
  s.   d.                              £
  7    6    :      1    3¾ loss  : :  100
 12               12
 ----            ----
 90d.            15d.
  4               4
 ----            ----
36,0 fgs .       (6) 630,0
 ----       36  {
              (6) 105
```

Here 7s. 6d. is in proportion to (1s. 3¾d.) its loss, as £100 is to its proportionate loss.

Answer, £17. 10s. per Cent.

Bought 12 Cwt. 2 qrs. 13 lb. of Sugar for £44. 3s. 1½d. ; at what rate must it be sold per lb. to gain £25 per Cent. ?

Cwt. qr. lb.	lb.	£ s. d.	
12 2 13	: 1 : :	44 3 1½	

1413 lb.

20

883 sgs.

12

1413) 10597 (7d.

..706

4

1413) 2826 (½

. . . .

First, the value of 1 lb. of Sugar is found to be 7½d.

£	£	d.
100	: 125	: : 7½
		5
4	5	
	4) 3s. 1½	

Answer,   9½+.5

Next, if £100 be advanced to £125, then also 7½d. must be proportionately advanced.

EXERCISES.

1. If Cheese is bought at £3. 14s. 8d. per Cwt., and sold at 10½d. per lb., what is gained per lb. ?   Answer, 2½d.

2. Bought 175 yards of Cloth at 5s. 4d. per yard, and sold them again at 7s. 6d. per yard: what was the gain on the whole ?

Answer, £18. 19s. 2d.

3. A merchant bought a quantity of Tobacco at 17½d. per lb. ; but not proving so good as he expected, sold it at 13¾d. per lb. : what was his loss per Cent. ?   Answer, £21. 8s. 6¾d.

4. If Raisins are bought at 15d. per lb., at what rate must they be sold to gain £25 per Cent. ?   Answer, 18¾d.

5. If when Cloth is sold at 8s. 9d. per yard, I gain 12 per Cent., what would be the gain per Cent., if sold for 10s. 6d. per yard ?

Answer, £34. 8s.

6. Sold 238 yards of Cloth at 4s. 7d. per yard, by which I gained £30 per Cent.: what did I gain in the whole?

Answer, £16. 7s. 3d.

7. Suppose Coffee is bought at 2s. 9d. per lb., at what rate must it be sold to gain £15 per Cent.? Answer, 3s. 1¾d.

8. Sold 12 Cwt. of Sugar for £43. 15s., and gained £20 per Cent.: what was the prime cost? Answer, £36. 9s. 2d.

9. If I buy Tobacco at 10 guineas per Cwt., at what rate must I retail it, per lb., to gain £12 per Cent.? Answer, 2s. 1¼d.

DUODECIMALS,

Or Cross Multiplication, is a Rule used by workmen and artificers in calculating the content of their work, the length and breadth being given in feet, inches, &c. It is performed by the following Table:—

            12 fourths ('''') .... 1 third .... 1'''
            12 thirds .... 1 second .... 1''
            12 seconds .... 1 inch .... 1'
            12 inches .... 1 foot .... 1 ft.

Example.

Multiply 7 feet, 6 inches, by 4 feet, 9 inches.

	ft.	′	
	7	6	
	4	9	
	30	0	
	5	7	6
Answer, Ft.	35	7	6

4 times 6 are 24; 24′=2 feet, carry 2. 4 times 7 and 2 are 30, 30 feet. Next, multiply by 9. 9 times 6 are 54.* 54 seconds=4′. 6″. Put 6″ down & carry 4′. 9 times 7 are 63 and 4 are 67. 67′= 5 ft. 7′.

* NOTE. Feet multiplied into feet; give feet.
Feet multiplied into inches, give inches.
Feet multiplied into seconds, give seconds.
Inches multiplied into inches, give seconds.
Inches multiplied into seconds, give thirds.
Seconds multiplied into seconds, give fourths.

Example.		
Ft.	in.	"
7	9	9
3	5	3

23	5	3		
3	3	0	9	
0	1	11	5	3

Answer, 26 10' 3" 2''' 3''''

By Practice.

	Ft.	in.	"		
	7	9	9		
			3		

	23	5	3		
4 in...⅓	2	7	3		
1 in...¼	0	7	9	9	
3"....¼	0	1	11	5	3

26 10' 3" 2''' 3''''

EXERCISES.

1. Multiply 3 ft. 6 in. by 2 ft. 7 in.—Answer, 9 ft. 0 in. 6"

2. Multiply 5 ft. 8 in. by 4 ft. 10 in.—Answer, 27 ft. 4 in. 8"

3. Multiply 7 ft. 10 in. by 5 ft. 3 in.—Answer, 41 ft. 1 in. 6"

4. Multiply 12 ft. 5 in. by 6 ft. 8 in.—Answer, 82 ft. 9 in. 4"

5. Multiply 24 ft. 3 in. by 10 ft. 11 in.—Answer, 264 ft. 8 in. 9"

6. Multiply 72 ft. 9 in. by 13 ft. 8 in.—Answer, 994 ft. 3 in.

7. Multiply 10 ft. 4 in. 5" by 7 ft. 8 in. 6"

Answer, 79 ft. 11' 0" 6''' 6''''

8. What is the price of a marble slab, whose length is 5 ft. 7 in., and breadth 1 ft. 10 in., at 6s. per foot ? Answer, £3. 1s. 5d.

9. What will the paving of a court-yard come to at 1s. 2d. per ft. the yard being 74 feet long, and 56 ft. 8 in. wide?

Answer, £244. 12s. 2¼d.

10. What is the charge for a piece of wainscoting at 6s. 7½d. per square yard, which measures 8 ft. 3 in. by 6 feet. 6 in. ?

Answer, £1. 19s. 5½d.

11. Required the solid content of a log of timber 36 ft. 6 in. long, 3 ft. 4 in. 2" broad, and 1 ft. 11 in. 3" thick ?

Answer, 236 ft. 8 in. 6"$\frac{7}{15}$.

COMPOUND INTEREST

Is that which is produced not only from the Principal lent, but also from the Interest, which is added to the Principal as it becomes due, making an increased Principal for each succeeding payment.

Example.

What is the compound interest of £700. 18s. for 4 years, at £5 per Cent. per annum?

	£	s.	d.	
	700	18	0	Principal.
£5=¹⁄₂₀	35	0	10¾	Interest for 1st year.
	735	18	10¾	Principal for 2d year.
£5=¹⁄₂₀	36	15	11¼	Interest for 2d year.
	772	14	10	Principal for 3d year.
£5=¹⁄₂₀	38	12	8¾	Interest for 3d year.
	811	7	6¾	Principal for 4th year.
£5=¹⁄₂₀	40	11	4½	Interest for 4th year.
	851	18	11¼	Amount for 4th year.
	700	18	0	Principal.
Answer,	151	0	11¼	Compound Interest.

RULE.

Find the interest for 1 year:—add this to the Principal;—find the Interest on this new Principal, and so proceed for the number of years required.

Note. If the payments are half-yearly, take half the rate, and twice the number of payments; if quarterly, take one-fourth of the rate, and four times the number of payments.

EXERCISES.

1. What is the compound interest of £500 for 3 years, at £4 per Cent. per annum?   Answer, £62. 8s. 7¼d.

2. What is the amount of £725 for 2½ years, at £5 per Cent. per annum, interest payable half-yearly?   Answer, £820. 5s. 4½d.

3. What is the compound interest of £387. 15s. for 2½ years, at £4 per Cent. per annum, interest payable quarterly?
          Answer, £35. 16s. 7½d.

`

ARITHMETICAL PROPORTION

AND PROGRESSION.

Arithmetical Proportion is the relation which quantities of the same kind have to each other, with respect to their differences.

Numbers are said to be in arithmetical progression when they increase or decrease by the repeated addition or subtraction of the same quantity. Thus, 2, 5, 8, 11, 14, &c. are an increasing arithmetical progression, the number 3 being repeatedly added; and 20, 16, 12, 8, &c. are a decreasing arithmetical progression, the number 4 being repeatedly subtracted.

If three numbers be in arithmetical progression, the sum of the extremes will be double of the mean. Thus in 4, 8, 12—(4+12)=(8×2).

If four numbers be in arithmetical progression, the sum of the extremes will be equal to the sum of the means. Thus in 3, 6, 9, 12—3+12 (extremes)=6+9 (means).

In arithmetical progression five things are to be considered, *viz.*

1. The first term ⎫
2. The last term ⎭ or extremes.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PROBLEM 1st.—The extremes and the number of terms being given, to find the sum of all the terms.

Example.

In an arithmetical progression the least term is 5, the greatest 205; and the number of terms 12. Required the sum of all the terms.

To   5 } extremes.
Add 205 }

          210
mult.   6 half No. of terms.

Answer, 1260 Sum of all terms.

RULE.—Add the extremes together, and multiply the sum by half the number of terms. The product will be the sum of all the terms.

EXERCISES.

1. If the least term be 3, the greatest 108, and the number of terms 14, what is the sum of the terms?   Answer, 777.

2. The extremes of an arithmetical progression are 4 and 200, and the number of terms 20. Required the sum of the terms.

Answer, 2040.

3. How many strokes does the hammer of a clock strike in 12 hours?   Answer, 78.

4. In Venice the clocks strike continually on to 24 o'clock; how many are the strokes in the compass of the day?   Answer, 300.

PROBLEM 2d. — The extremes and number of terms being given, to find the common difference.

Example.

The least term is 3, the greatest term 17, and the number of terms 8; what is the common difference?

No. of terms 8   17 greater.
             1   3 less.

          7   7)14

Answer,   2 com.dif.

RULE.—Subtract the less from the greater extreme, divide the remainder by 1 less than the number of terms, and the quotient will be the common difference.

G 5

EXERCISES.

5. The least term is 5, the greatest 27, and the number of terms 12: required the common difference. Answer, 2.

6. In an arithmetical series the extremes are 3 and 30, and the number of terms 10: what is the common difference? Answer, 3.

7. The extremes of an arithmetical series are 2 and 53, and the number of terms 18: required the common difference. Answer, 3.

8. A man had 8 sons; the youngest was 4 years old, and the eldest 32, and their ages were an arithmetical series: required the common difference. Answer, 4.

PROBLEM 3d.—The extremes and common difference being given, to find the number of terms.

Example.

If the least term be 3, the greatest 108, and 5 the common difference, required the number of terms.

$$\left.\begin{array}{r} 108 \\ 3 \end{array}\right\} \text{ extremes.}$$

com. dif. 5) 105

     21
     1 added.

Answer, 22 number of terms.

RULE.—Subtract the less extreme from the greater, and divide by the common difference; add 1 to the quotient, and the sum will be the number of terms.

EXERCISES.

9. In an arithmetical series the extremes are 4 and 39, and the common difference 5; required the number of terms. Answer, 8.

10. The least term is 7, the greatest 47, the common difference 2: required the number of terms. Answer, 21.

11. The extremes of an arithmetical series are 5 and 41, and the common difference 3: required the number of terms. Answer, 13.

12. If a traveller perform a journey by going 3 miles the first day, with an increase of 5 miles every day, until he travel 58 miles the last day: how many days will he travel? Answer, 12.

PROBLEM 4th.—The greatest term, the common difference, and the number of terms being given, to find the least term.

Example.

The greatest term is 84, the common difference 4, and the number of terms 21: required the least term.

$$\begin{array}{r} 21 \text{ No. of terms.} \\ 1 \\ \hline 20 \\ 4 \text{ common difference.} \\ \hline 80 \\ = \\ 84 \text{ greater term.} \\ 80 \text{ product.} \\ \hline \end{array}$$

Answer,   4 least term.

RULE.—Multiply the number of terms, less 1, by the common difference; subtract the product from the greatest term, and the remainder will be the least term.

EXERCISES.

13. The greatest term is 60, the common difference 3, and the number of terms 19: required the least term.   Answer, 6.

14. In a series of numbers in arithmetical progression the greatest term is 100, the number of terms 17, and the common difference 6: what is the least term?   Answer, 4.

15. The last term of an arithmetical series is 505, the number of terms 101, and the common difference 5: required the least term.

Answer, 5.

16. A person made 6 payments, of which the last was £28, increasing each payment £5: required the first sum paid.

Answer, £3.

PROBLEM 5th.—The number of terms, the common difference, and the sum of all the terms being given, to find the last term.

Example.

In a series of numbers in arithmetical progression, the number of terms is 21, the common difference 3, and the sum of all the terms 840: required the least term.

No. of
terms.

  21)840 Sum.   3 common difference.

     ——   20 No. of terms (less 1).

    40   ——

    30   2)60

    ——   ——

Ans. 10 least term. 30

RULE.—Divide the sum of all the terms by the number of terms, and from the quotient subtract half the product of the common difference multiplied by the number of terms, less 1; the remainder is the least term.

EXERCISES.

17. The number of terms is 11, the common difference 2, and their sum 154: required the least term.   Answer, 4.

18. In a series of numbers in arithmetical progression the number of terms is 17, the common difference 6, and the sum of the terms 884: what is the least term?   Answer, 4.

19. If the number of terms be 22, the common difference 5, and the sum of the terms 1221, what is the least term?   Answer, 3.

20. A debt of £135. 4s. is to be discharged in a year by weekly payments, increasing every payment 2s.: required the amount of the first payment.   Answer, 1s.

PROBLEM 6th.—The least term, the number of terms, and the common difference being given, to find the greatest term.

Example.

The first term is 6, the number of terms 20, and the common difference 8: required the greatest term.

          20 No. of terms.

           8 common differ.

          ——

         160

com. dif.   8 subtr.

         ——

        152

          6 first term.

Answer, 158

RULE.—Multiply the number of terms by the common difference, and subtract the common difference from the product; the remainder, added to the first term, gives the greatest term.

Exercises.

21. In an arithmetical progression the first term is 2, the number of terms 18, and the common difference 3: required the greatest term. *Answer*, 53.

22. The least term is 6, the number of terms 21, and the common difference 3: required the greatest term. *Answer*, 66.

23. If the least term be 3, the number of terms 22, and the common difference 5, what is the greatest term? *Answer*, 108.

24. A man bought 80 yards of Cloth: the first yard cost him 2s., and each succeeding yard 1s more, to the last: what did the last stand him in? *Answer*, £4. 1s.

GEOMETRICAL PROPORTION
AND PROGRESSION.

Geometrical Proportion is the ratio which quantities of the same kind have to each other, when multiplied or divided by the same quantity.

Numbers are said to be in geometrical progression when they increase or decrease by the repeated multiplication or division of the same quantity. Thus 2, 4, 8, 16, 32, &c. are an increasing geometrical progression, being repeatedly multiplied by two. And 81, 27, 9, 3, 1, are a decreasing geometrical progression, being repeatedly divided by 3.

If three numbers be in geometrical progression, the product of the extremes will be equal to the square of the means. Thus, in 3, 9, 27—3×27=9 squared.

Also, if three terms be in geometrical progression, the first term will have the same proportion to the second as the second has to the third.

If four numbers be in geometrical progression, the product of the extremes will be equal to the product of the means. Thus, in 4, 8, 16, 32—$(4\times32)=(8\times16.)$

Also, if four terms be in geometrical progression, the first term will have the same proportion to the second as the third has to the fourth.

In a geometrical progression consisting of any number of terms, the product of the extremes is equal to the product of any two terms equally distant from them, or to the square of the mean term, when the number of terms is odd. Thus, in the decreasing geometrical progression, 128, 64, 32, 16, 8, 4, 2 — $128\times2=64\times4=32\times8=16$ squared.

PROBLEM 1st. To find one geometrical mean proportional between any two numbers.

Example.

Required a geometrical mean proportional between 9 and 49.

$$\begin{array}{r} \text{Multiply } 49 \\ \text{by } 9 \\ \hline 441 \end{array}$$

Extract the square root $\overset{\bullet\;\bullet}{441}$ (21

$$\begin{array}{r} 4 \\ \hline 41)\;.41 \\ 41 \\ \hline \end{array}$$

Answer, 21, mean proportional.

RULE.—Multiply the two numbers together, and extract the square-root of the product, which will give the mean proportional sought.

EXERCISES.

1. Required a geometrical mean proportional between 3 and 12.

Answer, 6.

2. Required a geometrical mean proportional between 25 and 81.

Answer, 45.

3. What is the geometrical mean proportional between 27 and 243 ?

Answer, 81.

4. What is the geometrical mean proportional between 10 and 20 ?

Answer, 14. 14213, &c.

PROBLEM 2d. To find two geometrical mean proportionals between any two numbers.

Example.

Find two geometrical means between 5 and 625.

5) 625
_____
125
=====

Cube root of 125 is 5

Consequently 5×5= 25

and 25×5=125

Answer, 25 and 125.

RULE.—Divide the greater number by the less, and extract the cube root of the quotient, which will give the common ratio of the terms, which, being multiplied by the least given number, gives the first mean, and this multiplied by the ratio, gives the second mean.

EXERCISES.

5. Find two geometrical means between 4 and 256.

Answer, 16 and 64.

6. Find two geometrical means between 8 and 512.

Answer, 32 and 128.

7. Find two geometrical means between 9 and 1125.

Answer, 45 and 225.

8. Find two geometrical means between 15 and 5145.

Answer, 105 and 735.

PROBLEM 3d. The first term, the ratio, and the number of terms being given, to find the last or any other term.

Example.

If the first term be 2, the ratio 2, and the number of terms 19, what is the last or greatest term?

```
        2 ratio.
        2
        ─
        4 2d power.
        4
        ─
       16 4th power.
       16
      ────
      256 8th power.
      256
      ────
     1536
     1280
      512
    ──────
    65536 16th power.
        4
    ──────
   262144 18th power.
        2
    ──────
```
Answer, 524288 last term.

RULE. Find such a power of the ratio as is denoted by the number of terms less one, and the result multiplied by the first term, will give the term required.

Here, the number of terms is 19, and 19 less 1 = 18. Then, the 18th power is multiplied by the first term.

EXERCISES.

9. In a series of numbers in geometric progression, if the first term be 3, the ratio 2, and the number of terms 12, required the last term.   Answer, 6144.

10. If I buy 12 Oranges, paying one farthing for the first, one half-penny for the second, a penny for the third, and so on, doubling the price of each, what would the last come to?   Answer, £2. 2s. 8d.

11. A Draper sold 20 yards of Cloth; the first yard for 3d., the second for 9d., the third for 27d., &c., increasing in geometrical progression: what was the last yard sold for?

Answer, 3486784401 pence.

LOGARITHMS.

A Logarithm is the index, or exponent of a power to which a certain or invariable number must be raised, in order to equal the common natural number.

The invention of Logarithms is due to Lord Napier, Baron of Merchiston, in Scotland, who first published the Table in 1614; but Mr. Briggs, then Professor of Geometry at Gresham College, improved and reduced them to a more convenient form, which is now generally used.

The Tables are made use of to facilitate tedious arithmetical calculations.

They are founded upon a series of numbers in geometric progression, increasing in a tenfold proportion.

Thus $\begin{cases} 0. \quad 1. \quad 2. \quad 3. \quad 4. \quad\quad 5. \quad\quad \text{Indices, or Logarithms.} \\ 1. \quad 10. \quad 100. \quad 1000. \quad 10000. \quad 100000. \quad \text{Geometric Progression.} \end{cases}$

The distinguishing feature of this system of Logarithms is, that the index or logarithm of 10 is 1, that of 100 is 2, that of 1000 is 3, that of 10,000 is 4, &c. Therefore the logarithm of any number between 1 and 10 will be 0 and some decimal parts, and that of a number between 10 and 100 will be 2 and some decimal parts, and so on.

Hence the integral part of any logarithm, which is usually called its index, is always *less by one* than the number of integers which the natural number consists of; and for decimals, it is the number denoting the distance of the first significant figure from the place of units, as seen in the following page.

Natural Numbers.		Indices.
3	=	0. and decimal parts.
78	=	1. and decimal parts.
3256	=	3. and decimal parts.
.038	=	$\overline{2}$. and decimal parts.
.00294	=	$\overline{3}$. and decimal parts.
.521	=	$\overline{1}$. and decimal parts.

Logarithms are used in abridging the operations of multiplication and division, in the raising and extracting of roots, and in the proportion of decimal fractions.

DIRECTIONS FOR USING A TABLE OF LOGARITHMS.*

If the given number be less than 100.

The logarithm is immediately found by inspection of the Table, which contains all numbers from 1 to 100, with their logarithms and indices annexed.

<p style="text-align:center">The Log. of 6 is 0.7781513.</p>
<p style="text-align:center">„ „ 97 is 1.9867717.</p>

Here 0 is placed as the index to the logarithm of 6, because it is less than 10. And 97 has 1 for an index to the logarithm, being more than 10, and less than 100.

If the given number contain 3 figures :—

Find the logarithm by inspection, as before, to which prefix 2 for an index.

<p style="text-align:center">Thus—Log. of 365 is 2.5622929.</p>

---

* In the use of the Logarithms, these directions have reference to Dr. Hutton's Mathematical Tables. A smaller book of Tables, by Mr. J. R. Young, comprehending the Logarithms of all numbers from 1 to 36,000, may be found useful to the student.

If the given number contain 4 figures :—

Look for the figures of the given number in the first column of the page (marked with N above), and the logarithm is in the next column.

Thus, the Log. of 5379 in the Table is 7307015, to which 3 is prefixed for an index.—Log. of 5379=3.7307015.

If the given number contain 5 figures.

Find the first 4 figures, as before, and the fifth at the top of the page, under which, in the same line with the first four figures, stand the last 4 figures of the logarithm, the first three of which must be sought from the column marked 0.

Thus, the Log. of 72835 in the Table is 8623401, to which 4 is the index.—Log. of 72835=4.8623401.

If the given number contain 6, or more figures :—

Find the logarithm of the first 5 figures, as before; then multiply the common tabular difference* by the remaining figures, cutting off from the product as many figures as you multiplied by, and add the remaining figures to the logarithm of the first five.

To find the log. of 46.7512 :—

The log. of the first five (46751)=6697909 by the Table: the remaining figure (2) multiplied by the tabular difference   93
                                                                                    2
                                                                            ———————
                                                                              18.6

$$
\begin{array}{llll}
\text{Log. of } 46.751 & = & 6697909 \\
\text{,, \quad ,,} \qquad 2 & = & 18 \\
\hline
\text{Log. of } 46.7512 & = & 1.6697927
\end{array}
$$

* The tabular difference is in the column on the right-hand of the page, under that marked D.

Again—To find the log. of 32728696 :—

Log. of 32728...	=	5149195	133 tab. diff.
696	=	92	696
Log. of 32728696	=	7.5149287	2088
			9048
			92,568

The Logarithms of Decimals always have the minus (—) sign placed over the indices, to distinguish them from the logarithms of whole numbers, which are consi- dered (though not expressed) to have the positive (+) sign prefixed.

The Logarithms of Decimals are to be found as in whole numbers, and the indices are placed according to their distances from the decimal point, as before referred to.

If one place from the decimal point, the index will be —1, if two places—2, &c.

	Numbers.		Logarithms.
Thus,	.2176	=	$\bar{1}$.3376589
	.05369	=	$\bar{2}$.7298934
	.0063	=	$\bar{3}$.7993405
	.000279	=	$\bar{4}$.4456042.

The logarithm of any number, whether whole or deci- mal, consists of precisely the same figures, differing only in the value of the index, as follows :—

Numbers.		Indices Logarithms.
72538	=	4.8605656
7253.8	=	3.8605656
725.38	=	2.8605656
72.538	=	1.8605656
7.2538	=	0.8605656
.72538	=	$\bar{1}$.8605656
.072538	=	$\bar{2}$.8605656
.0072538	=	$\bar{3}$.8605656

To find the natural number corresponding to a given logarithm.

This is to be found by the reverse method to the former. Suppose it is required to find the natural number answering to the logarithm 3.2946646.   Find the first three figures (294) after the index in the column of the Tables marked 0 ; and the other figures (6646) in the same line in one of the following columns. If the figures of the logarithm be thus found exactly, then on the same line in the first column are the first 4 figures (1970) of the natural number, and the other figure (9) is at the top of that column in which the last 4 figures (6646) were found.

And as the given logarithm has 3 for its index, by reversing the directions given to determine the index, four integers (1970) are pointed off for whole numbers.

$$3.2946646 = 1970.9 \text{ natural number.}$$

If the figures in the given logarithm cannot be found exactly, find the next less logarithm in the Table, and subtract it therefrom ; divide the remainder by the common tabular difference, and the quotient will be the additional figures beyond five.

Required the natural number of log. 2. 3720568.

```
Given log. is ................2. 3720568
The next less log. in the Table is    3720462 = nat. num. 23553.
                                      ———————
            Difference    ....106
                          ══════

      Tab. Diff. 185) 106.00 (57, &c.
                      —————
                      1350        2355357
                      ————
                      ..55
                      ════
```

The given log. having 2 for its index, 3 integers must be pointed off for whole numbers.

Log. 2. 3720568 = 235.5357 nat. number.

MULTIPLICATION BY LOGARITHMS.

Rule.—Find the logarithms of the factors, and add them together, and their sum will be the logarithm of the product required. Then find the natural number corresponding to the product.

Examples.

Multiply 325.89 with 72.85 by logarithms.

Numbers.		Logarithms.
325.89	=	2.5130710
72.85	=	1.8624296
Product 23741.0865	=	4.3755006

Multiply 7.51863 with .2376 by logarithms.

Numbers.		Logarithms.	
7.51863	=	0.8761372	Here $+ 1$ to be carried from the decimals, cancels—1; consequently 0 is set down from the top line.
.2376	=	$\overline{1}$.3758464	
Product 1.7864201, &c.=		0.2519836	

Multiply 2.316—40879.1—and .00872 together by logarithms.

Numbers.		Logarithms.	
2.316	=	0.3647386	Here $+ 1$ carried from the decimals, added to $+ 4 = + 5$; and $- 3$ subtracted therefrom, leaves $+ 2$ in the total.
40879.1	=	4.6115014	
.00872	=	$\overline{3}$.9405165	
Product 825.574, &c. =		2.9167565	

Multiply 4.2293—.0174—.0058—and 9073.6 together by logarithms.

Numbers.		Logarithms.	
4.2293	=	0.6262685	Here $+ 2$ carried from the decimals, cancels $- 2$. Also $+3$ cancels -3; consequently 0 is put down in the total.
.0174	=	$\overline{2}$.2405492	
.0058	=	$\overline{3}$.7634280	
9073.6	=	3.9577796	
Product 3.872801, &c. =		0.5880253	

1. Multiply 70.638 with 5.764, by logarithms.

Product 407.1574.

2. Multiply 827.603 with .7253, by logarithms.

Product 600.2604.

3. Multiply 984.653 with 7.64215, by logarithms.

Product 7524.865.

4. Multiply 5.23861 with .00754, by logarithms.

Product .0394991.

5. Multiply 52.735—9.076, and 384.3 together, by logarithms.

Product 183934.7.

6. Multiply 96503.8—3.0027—.1284, and 472.807 together, by logarithms.   Product 17591597.

DIVISION BY LOGARITHMS.

RULE. Subtract the logarithm of the divisor from the logarithm of the dividend, and the natural number, answering to the remainder, is the quotient required.

Examples.

Divide 47965 by 5642, by logarithms.

Numbers.		Logarithms.
47965	=	4.6809244
5642	=	3.7514331
Quotient 8.50141	=	0.9294913

Divide 18.276·by 7.2412, by logarithms.

Numbers.		Logarithms.
18.276	=	1.2618811
7.2412	=	0.8598105
Quotient 2.52389	=	0.4020706

Divide 378.627 by 490.53, by logarithms.

Numbers.		Logarithms.
378.627	=	2.5782117
490.53	=	2.6906656
Quot. .771873	=	$\overline{1}$.8875461

If the subtraction cannot be made as usual, add the 1 that is to be carried from the decimal part (when it occurs) to the index of the logarithm of the divisor; *conceive the sign changed*, and put down the difference of the indices, either+ or —, as the remainder.

Here 1, carried to the lower index, and supposed to have the sign changed, is — 3. Then the difference between +2 and — 3, is — 1.

Divide .0216583 by .0047219, by logarithms.

Numbers.		Logarithms.
.0216583	=	$\overline{2}$.3356243
.0047219	=	$\overline{3}$.6741168
Quo. 4.58677	=	0.6615075

Here+1 carried to — 3 makes — 2. And supposing—2 changed into+2, then the difference between—2 and+2 is 0.

EXERCISES.

1. Divide 40726 by 7581 by logarithms.   Answer, 5.37211.

2. Divide 5.29871 by .8176 by logarithms.   Answer, 6.480809·

3. Divide 3728.649 by 157.38 by logarithms.   Answer, 23.69201.

4. Divide 215.8796 by 9.7216 by logarithms.   Answer, 22.20618.

5. Divide 3.2709 by .22158 by logarithms.   Answer, 14.76171.

6. Divide .030765 by .004288 by logarithms.   Answer, 7.17467.

THE RULE OF THREE IN LOGARITHMS.

The application of the Rule of Three in Logarithms will be seen in Plane Trigonometry, and other branches of Mathematics.

INVOLUTION BY LOGARITHMS.

Take out the logarithm of the given number from the Table, and multiply it by the index of the proposed power; and the number answering to the product, will be the power required.

Examples.

Find the square of 5.7209 by logarithms.

	Number.		Logarithm.
Root	5.7209	=	0.7574644
		Index	2
Square ..	32.72869, &c. =		1.5149288

Required the cube of 79.0568 by logarithms.

	Number.		Logarithm.
Root	79.0568	=	1.8979392
		Index	3
Cube....	494103.06, &c. =		5.6938176

Find the cube of .053406 by logarithms.

	Number.	Logarithm.	
Root ..	.053406	= $\bar{2}$.7275901	Here the index is—2, which, when multiplied, is—6; but as + 2 are to be carried from the decimals, it leaves — 4, as the index of the product.
		Index 3	
Cube	.0001523247, &c.	= $\bar{4}$.1827703	

Find the 4th power of .076543 by logarithms.

	Number.	Logarithm.	
Root..	.076543	= $\bar{2}$.8839055	Here also the index is — 2, which, when multiplied, is—8; but as + 3 are to be carried from the decimals, it leaves — 5, as the index of the product.
		Index 4	
4th power	.000034325, &c. =	$\bar{5}$.5356220	

EXERCISES.

1. Find the square of 3.1879 by logarithms.

Answer, 10.1627, &c.

2. Required the cube of 9.2134 by logarithms.

Answer, 782.095, &c.

H

3. Required the square of .72536 by logarithms.

Answer, .526147, &c.

4. Involve .37216 to the 4th power by logarithms.

Answer, .0191831, &c.

5. Find the cube of .20793 by logarithms.

Answer, .0089898, &c.

6. What is the 5th power of 1.0764 by logarithms ?

Answer, 1.445002, &c.

EVOLUTION BY LOGARITHMS.

Take the logarithm of the given number out of the Table. Divide the logarithm by 2 for the square root, 3 for the cube root, &c.; and the number answering to the quotient will be the root.

Examples.

Find the square root of 802.95 by logarithms.

	Number.		Logarithm.
	802.95	=	2.9046885
			2) ————
Root	28.3363, &c.	=	1.4523442

Find the cube root of 5735339 by logarithms.

	Number.		Logarithm.
	5735339	=	6.7585590
			3) ————
Root	179	=	2.2528530

Required the cube root of 15.7286 by logarithms.

	Number.		Logarithm.
	15.7286.	=	1.1966901
			3) ————
Root	2.50551, &c.	=	0.3988967

What is the 4th root of 571.914 by logarithms?

	Number.		Logarithm.
	571.914	=	2.7573307
			4)———
Root	4.89026, &c.	=	0.6893326

EXERCISES.

1. Find the square root of 368863 by logarithms.

Answer, 607.34, &c.

2. Required the cube root of 48228544 by logarithms.

Answer, 364.

3. What is the cube root of 15926.9 by logarithms?

Answer, 25.1596, &c.

4. What is the cube root of 33.230979 by logarithms?

Answer, 3.21502, &c.

5. Required the 4th root of 93.217 by logarithms.

Answer, 3.10723, &c.

6. What is the square root of 85.9333 by logarithms?

Answer, 9.27002, &c.

7. Required the cube root of 36155.02 by logarithms.

Answer, 33.0666, &c.

8. What is the cube root of 9.166 by logarithms?

Answer, 2.09279, &c.

MENTAL ARITHMETIC.

1. To find the value of a dozen articles.

RULE.—Take the price in pence as shillings, and for every farthing add 3 pence to the value.

Example 1.—12 lb. of Meat at 8d. per lb.
   The price in pence, taken as shillings, is 8 shillings.

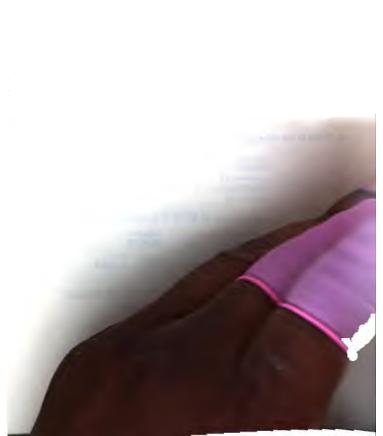*Example* 2.—A dozen yards of Tape, at 5½d per yard.
   5 pence, taken as 5s., to which add 6d. for the ½d., gives the value, 5s. 6d.

Example 3.—How much will a dozen lb. of Coffee come to, at 2s. 8¾d. per lb.?
   2s. 8d. = 32d., taken as shillings, 32s.; and 9d. added thereto for ¾d. = 32s. 9d. = £1. 12s. 9d.

EXERCISES.

1. 12 lb. of moist Sugar, at 7d. per lb.
2. 12 lb. of Loaf Sugar, at 11d per lb.
3. 12 yards of Cotton, at 14d per yard.
4. 12 oz. of Nutmegs at 9d. per oz.
5. 12 lb. of Beef, at 10¼d. per lb.
6. 12 pints of Wine, at 1s. 2½d. per pint.
7. 12 lb. of Wax Candles, at 1s. 5¼d. per lb.
8. 12 lb. of Coffee, at 2s. 3d. per lb.
9. A dozen pair of Gloves, at 3s. 7½d. per pair.
10. A dozen of Wine, at 4s. 3¾d. per bottle.
11. A dozen pairs of Shoes, at 6s. 7½d. per pair.
12. A dozen Legs of Mutton, at 9s. 10¾d. each.

MENTAL ARITHMETIC.

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1. To find the value of a dozen articles.

RULE.—Take the price in pence as shillings, and for every farthing add 3 pence to the value.

Example 1.—12 lb. of Meat at 8d. per lb.
The price in pence, taken as shillings, is 8 shillings.

Example 2.—A dozen yards of Tape, at 5¼d. per yard.
5 pence, taken as 5s., to which add 6d. for the farthing, is the value, 5s. 6d.

Example 3.—How much will a dozen lb. of Tea, at 8¾d. p

.ce,

.d., which,

, which, taken
£5. 1

2. To find the value of any number of Dozens.

RULE.—Find the value of one dozen, and multiply by the given number.

Example 1.—4 dozens of Slates, at 8½d. each.

　Calculated mentally thus:—1 dozen, at 8½d., is 8s. 6d., (by the preceding Rule), and 8s. 6d. multiplied by 4=£1. 14s., the value.

Example 2.—60 pairs of Stockings, at 2s. 3d. per pair.

　1 dozen at 2s. 3d.=27d.,=27s. and 27s.×5=135s.=£6. 15s., the value.

Example 3.—132 yards, at 3s. 4½d. per yard.

　1 dozen at 40½d.=40s. 6d., and £2. 0s. 6d. multiplied by 11 =£22. 5s. 6d.

<div align="center">EXERCISES.</div>

　13. 24 yards of Ribbon, at 5d. per yard.

　14. 4 dozens of Books, at 8d. each.

　15. 72 Lemons, at 2½d. each.

　16. 5 dozens of Inkstands, at 3¼d. each.

　17. 8 dozens of Copy-books, at 9d. each.

　18. 36 pints of Oil, at 1s. 10½d. per pint.

　19. 84 lb. of Sugar, at 7¼d. per lb.

　20. 9 dozens of Pigeons, at 14¾d. each.

　21. 96 lb. of Tobacco, at 1s. 8½d. per lb.

　22. 132 bushels of Oats, at 1s. 3d. per bushel.

　23. 10 dozens of Knives, at 9½d. each.

　24. A gross of Rulers, at 16d. each.

3. To find the value of a Score.

RULE.—If the price be shillings only, take the shillings as pounds.

Example 1.—20 books at 6s. each=£6.

Example 2.—A score of yards, at 17s. each=£17.

If the price contain pence also, multiply the pence by 20, and find the shillings, which add to the pounds.

Example 1.—20 yards, at 7s. 3d. a yard=£7. 5s.

Mentally found thus:—7s. taken as £7., and 20 threepences =60d.=5s.=£7. 5s.

Example 2.—A score of Penknives, at 3s. 5d. each=£3. 8s. 4d.

Thus:—3s. taken as £3, and 20 fivepences = 100d. = 8s. 4d. £3. 8s. 4d.

Example 3.—A score of Ducks, at 4s. 9d. each.=£4. 15s.

Thus:—4s. taken as £4, and 20 times 9=180d.=15s.=£4. 15s.

EXERCISES.

25. 20 yards, at 3s. per yard.
26. 20 dozens at 8s. per dozen.
27. 20 days' labour at 4s. 6d. per day.
28. 20 lb. of Tea, at 6s. 2d. per lb.
29. A score of Geese, at 9s. 5d. each.
30. 20 Hogsheads of Beer, at 28s. per hogshead.
31. 20 gallons of Wine at 4s. 3d. per gallon.
32. A score of Turkeys, at 17s. 5d. each.
33. 20 yards of Lace, at 23s. 6d. per yard.
34. 20 stone of Cheese, at 8s. 9d. per stone.
35. A score of Books, at 12s. 1½d. each.
36. A score of Inkstands, at 1s. 3¾d. each.

4. To find the value of any number of Grosses.

RULE.—Find the value of 1 dozen, and take that amount, in pence, as the value of another dozen.

Example 1.—1 gross at 2¼d. each.

1 dozen at 2¼d. each, is 2s. 3d., and 2s. 3d. = 27d., which, taken as shillings=£1. 7s., the value.

Example 2.—3 gross, at 9½d. each.

1 dozen, at 9½d. is 9s. 6d., and 9s. 6d.=114d., which, taken as shillings=£5. 14s. for one gross; and 3 times £5. 14s. =£17. 2s., the value.

Or, Take the number of farthings as shillings, and multiply by 3.

1st Example proved.—1 gross at 2¼d. each.

  2¼d. = 9 farthings, taken as shillings, and multiplied by 3=
    27s.=£1. 7s., the value.

2d. Example proved.—3 gross, at 9½d. each.

  9½d.=38 farthings, taken as shillings, and multiplied by 3=
    114s. ; then £5. 14s.×3=£17. 2s.

EXERCISES.

37. 1 gross, at 1¼d. each.	43. 1 gross. at 11d. each.
38. 1 gross, at 6d. each.	44. 1 gross, at 15½d. each.
39. 1 gross, at 4¼d. each.	45. 2 gross, at 10d. each.
40. 1 gross, at 8½d. each.	46. 5 gross, at 8¼d. each.
41. 1 gross, at 10¾d. each.	47. 3 gross, at 13d. each.
42. 1 gross, at 7¾d. each.	48. 4 gross, at 1¾d. each.

5. To find the value of any quantity, when the former Rules are not easily applied.

RULE.—Take the articles as pence, and multiply by the money.

Example 1.—29 lb. of Sugar, at 9d. per lb.
        29 lb., taken as pence=2s. 5d.
        and multiplied by   9
                     ——————
                   21s. 9d. value.

Example 2.—75¼ lb. of meat, at 10½d per lb.
      75¼ lb., taken as pence=6s. 3¼d.
      and multiplied by   10½
                 ————————
              62 11
         ¼   3 1¾
               ————————
            66s. 0¾d. value.

Example 3.—38¼ yards, at 7s. 3d. per yard.
      38¼, taken as shillings=38s. 3d.
      and multiplied by   7¼
                267 9
      3d. is ¼   9 6¾
             ——————————
       £13. 17s. 3¾d. value.

NOTE. If the price be in shillings, and either the quantity or price an even number, multiply, mentally, by half the even number, doubling the first product for shillings, and the rest will be pounds.

Thus, 43lb. at 14s., is known by multiplying, mentally, the odd number 43, by 7 (half the even number 14). 7 times 3 doubled=42 =£2. 2s., and 7 times 4=28. £28+£2. 2s.=£30. 2s., the value.

Again, the value of 34 gallons, at 29s. is known by multiplying, mentally, 29 by 17 ; thus 17 times 9=153, doubled=306s.=£15. 6s. and 17 times 2=£34, to which add £15. 6s.=£49. 6s., the value.

EXERCISES.

49. 43 lb. of Sugar, at 8d. per lb.

50. 29 lb. of Iron, at 2¼d. per lb.

51. 73 dwts. of Silver, at 7½d. per dwt.

52. 51 bushels, at 14d. per bushel.

53. 87 lb. of Cheese, at 5½d. per lb.

54. 23¼ yards, at 4d. per yard.

55. 79½ lb. of Soap, at 9d. per lb.

56. 38¼ gallons, at 3½d. per gallon.

57. 44 lb. of Tea, at 6s. per lb.

58. 91 yards, at 16s. per yard.

59. 117½ pints, at 1s. 4d. per pint.

60. 181¾ lb., at 2s. 1¼d. per lb.

APPLICATION OF THE FOREGOING RULES,

SHEWING THE MENTAL PROCESS IN CALCULATING BILLS, &c.

NOTE. It would be impossible to give a particular Rule for each separate calculation. A few general Rules have been given; but their application to business must be left to the invention and understanding of the calculator.

A GROCER'S BILL.

London, April 8, 1844.

Mr. Thomas Andrews,

Bought of Thomas Jones.

			d.			£	s.	d.
12	lb. of moist Sugar .... at		8½ per lb...			0	8	6
37	lb. of Loaf Ditto .... at	10		,,	..	1	10	10
21¼	lb. of Raisins.................... at	7		,,	..	0	12	6½
¼ Cwt. of Coffee .... at 2/7				,,	..	3	12	4
15	lb. of Muscatel Raisins........... at 1/1½			,,	..	0	16	10½
10¼	lb. of Souchong Tea............. at 6/8			,,	..	3	10	0
25	lb. of Currants................. at	4		,,	..	0	8	4
					£10	19	5

12 lb. of Sugar at 8½d.=8s. 6d. This is instantly known by Rule 1st.

37 lb. at 10d.=30s. 10d. Mentally known, thus—37 lb., taken as 37d.=3s. 1d. (Rule 5), and 10 times 3s. 1d.=30s. 10d., the value.

21¼ lb. at 7d.=12s. 6½d. Consider 21¼ lb. as 21¼d.; and 7 times 21¼d.=150¼d.=12s. 6½d., the value.

¼ Cwt.=28 lb. at 2s. 7d.=£3. 12s. 4d. Say 28 lb. at 2s. 6d. =28 Half Crowns=£3. 10s. Also 28 lb. at 1d.=28d.=2s. 4d. £3. 10s. and 2s. 4d.=£3. 12s. 4d., the value.

15 lb. at 1s. 1¼d.=16s. 10½d. Mentally found, thus—15 lb. at 1s.=15s.; to which add ¼ (1¼d.=¼ of a shilling); ¼ of 15s.= 1s. 10½d. 15s. and 1s. 10½d.=16s. 10½d., the value.

10¼ lb. at 6s. 8d.=£3. 10s. Thus, 6s. 8d. being ⅓ of a pound, ⅓ of £10=£3. 6s. 8d., to which add 3s. 4d.=value of ½ lb., gives £3. 10s., the amount.

25 lb. at 4d.=8s. 4d. Say, 4d. being ⅓ of a shilling; ⅓ of 25s. =8s. 4d., which is the value.

When the Bill has been paid, the Receiver, Thomas Jones, gives an acknowledgment to the Payer, Thomas Andrews, written upon a

6d. stamped paper (the value being above £10 and less than £20*),
as follows:—

London, May 7, 1844.

Received of Mr. Thomas Andrews, ten Pounds, nineteen Shillings,
and five Pence, the amount of the Bill delivered.

THOMAS JONES.

£10 : 19 : 5

A CHEESEMONGER'S BILL.

Stepney, April 26, 1844.

William Smith, Esq.

Bought of John Peters.

	d.		£	s.	d.
7 lb. of Bacon .... at 11½ per lb...			0	6	8½
18 lb. of Cheshire Cheese.............. at 9	„	..	0	13	6
11 lb. of Stilton Ditto .... at 14	„	..	0	12	10
½ Firkin of Butter, weight 56 lb........ at 13½	„	..	3	3	0
A fine Ham, weight 16 lb.............. at 1/7	„	..	1	5	4
23 lb. of fresh Butter.................. at 1/3	„	..	1	8	9
38 lb. of Gloucester Cheese............ at 10	„	..	1	11	8
			£9	1	9½

7 lb. of Bacon at 11½d.=6s. 8½d. 7 lb. at 1s., less 7 half-pence
(3½d.)=7s. less 3½d.=6s. 8½d., the value.

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* FOR RECEIPTS.

If for £5, and under £10, the Stamp is......	0s. 3d.
£10 „ £20 „	0s. 6d.
£20 „ £50 „	1s. 0d.
£50 „ £100 „	1s. 6d.
£100 „ £200 „	2s. 6d.

&c. &c. &c.

18 lb. at 9d.=13s. 6d.   1 dozen at 9d. is 9s., and half-a-dozen =4s. 6d.   9s. and 4s. 6d.=13s. 6d., the amount.

11 lb. at 14d.=12s. 10d.   11 times 14=154d.=12s. 10d., the value.

56 lb. at 13½d.   Say 56 lb. at 1s.=56s., and ¼ of 56 (for the 1¼d.)=7s.   56 and 7=63s.=£3. 3s.

16 lb. at 1s. 7d.=£1. 5s. 4d.   16 lb. at 1s.=16s., and 16 lb. at 7d.=112d.=9s. 4d.   16s.+9s. 4d.=£1. 5s. 4d., the amount.

23 lb. at 1s. 3d.=£1. 8s. 9d.   23 lb. at 1s.=23s., and 23 lb. at 3d.=69d.=5s. 9d.   23s. and 5s. 9d.=28s. 9d.=£1. 8s. 9d., the value.

38 lb. at 10d.=38s. all but 38 twopences=6s. 4d.   38s. less 6s. 4d.=31s. 8d.=£1. 11s. 8d., the value.

Or, 38 lb. taken as 38d.=3s. 2d.   Then, 10 times 3s. 2d.= 31s. 8d.=£1. 11s. 8d.

A BUTCHER'S BILL.

Clapham, April 18, 1844.

John Edwards, Esq.

To Timothy Brown.

				d.		£.	s.	d.
Boiling Beef........weight 15½ lb. .... at	7	per lb. ..	0	9	0½
Sirloin of Ditto ...... „	8 lb. 5 oz.. at 10½	„	.. 0	7	3
Veal............... „	39 lb...... at 14	„	.. 2	5	6
2 Legs of Mutton .... „	19½ lb..... at 8	„	.. 0	13	2
Saddle of Ditto ...... „	10 lb. 6 oz.. at 9½	„	.. 0	8	2½
3 Legs of Pork ...... „	21¼ lb..... at 10½	„	.. 0	18	7
Quarter of Lamb...... „	9 lb 14 oz.. at 11¼	„	.. 0	9	3
			£5	11	0

15½ lb. at 7d. = 9s. 0½d.   Mentally found, thus—Consider 15½ lb. as pence, 15½d.=1s. 3½d.×7=9s. 0½d.   Or, say 1 dozen lb. at 7d.=7s., and 3½ lb. at 7d.=2s. 0½d.; to which add 7s.= 9s. 0½d. the value.

8 lb. 5 oz. at 10½d.=7s. 3d.   Thus, 8 lb. at 10½d.=84d.=7s. —5 oz. = ¼ of a lb. and 1 oz.; ¼ of a lb. at 10½d. = 2⅝d., and ¼ part of 2⅝d. (for the 1 oz.)=⅝d.   Add, mentally, 7s., 2⅝d., and ⅝d. =7s. 3d.

39 lb. at 14d. = £2. 5s. 6d.   39 lb. at 1s.=39s., and 39 two-pences = 78d. = 6s. 6d.; 39s. and 6s. 6d. = 45s. 6d.=£2. 5s. 6d., the value.

19¾ lb. at 8d. = 13s. 2d.   20 lb. at 8d. = 160d. = 13s. 4d., from which subtract 2d. (for the ¼ lb. less than 20 lb.)=13s. 2d.

10 lb. 6 oz. at 9¼d.=8s. 2¼d.   10 lb. at 9¼d.=95d.=7s. 11d. 6 oz.=⅜ of a lb. and ⅛ of a lb.; ¼ of a lb. at 9¼d.=2¼d., and ⅛ of 9¼d.=1⅛d.   Add 7s. 11d., and 2¼d., and 1⅛d., mentally=8s. 2¼d., the value.   Or, the 6 oz. may be found thus—6 oz.=₁⁶₆=⅜ of a lb.; ⅛ of 9¼d.=1⅛d.; and 3 times 1⅛d.=3⅜d.   Observe, in calcu-lating ounces, &c., the value is seldom ascertained to an exact farthing.

21¼ lb. at 10½d. = 18s. 7d.   21¼ lb. as pence=1s. 9½d. × 10 = 17s. 8½d.   For the ½d. take ½ of 21¼d. = 10½d.   Then add 17s. 8½d. and 10½d.=18s 7d., the value.

9 lb. 14 oz. at 11½d.=9s. 3d.   Thus 10 lb. × 11½d.=9s. 4½d. but 9 lb. 14 oz. = 10 lb. less 2 oz., and 2 oz. = ⅛ of a lb., conse-quently ⅛ of 11½d.=1½d. nearly.   Then 9s. 4½d., less 1½d.=9s. 3d., the value.

A LINEN DRAPER'S BILL.

London, May 5, 1844.

Mrs. Oliver,

Bought of Henry White.

	d.		£	s.	d.
61 yards of Irish................at 3/11½ per yard....			12	1	5½
100 yards of Tape...............at	2¼	„	0	18	9
19 yards of Silk ....at 5/7		„	5	6	1
142 yards of Cotton ....at 2/1½		„	15	1	9
35¼ ells dimity.................at 1/9 per ell			3	2	6¼
21½ yards of Sheeting...........at 3/3 per yard....			3	9	10½
			£40	0·	5¼

61 yards at 3s. 11½d.=£12. 1s. 5½d.   Calculated mentally, thus— 3s. 11½d.=4s. less one halfpenny.   61 yards at 4s.=244s.=£12. 4s.,

and 61 yards at ½d.=61 halfpence=30½d.=2s. 6½d.   Then £12. 4s. less 2s. 6½d.=£12. 1s. 5½d., the value.

100 yards at 2¼d =18s. 9d.   100 yards at 1d.=100d. × 2= 200d., and ¼ of 100d.= 25d. (for the farthing) 225d.=18s. 9d., the value.

19 yards at 5s. 7d.=£5. 6s. 1d. 19=1 score, less 1. By Rule 3, 1 score at 5s. 7d=£5. 11s. 8d., from which deduct 5s. 7d. for 1 yard=£5. 6s. 1d.

142 yards at 2s. 1½d.=£15. 1s. 9d.   142=1 gross, less 2.   By Rule 4, 1 dozen at 2s. 1½d.=25s. 6d., and 12 times 25s. 6d. = 306s. =£15.6s., from which deduct 4s. 3d. (value of 2 yards)=£15.1s.9d., the value.   Or, 142 yards × 2s. =284s.=£14. 4s; and 142 yards at 1½d.=142÷8=17s. 9d.   £14. 4s. and 17s. 9d.=£15. 1s. 9d., the value.

35¾ yards at 1s. 9d.=£3. 2s. 6¾d.   35¾ = 3 dozen minus ¼. 1 dozen at 1s. 9d.=21s., and 3 dozen, 63s.; then ¼ of 1s. 9d.=5¼d., which subtracted from 63s., gives 62s. 6¾d.=£3. 2s. 6¾d., the value.

21½ yards at 3s. 3d.=£3. 9s. 10½d.   A score at 3s. 3d. = £3. 5s., and 1½ yards more=3s. 3d., and half of 3s. 3d.=1s. 7½d. =4s. 10½d., to which add £3. 5s.=£3. 9s. 10½d., the value.   Or, 21 yards at 3s.=63s.; 21 yards at 3d.=5s. 3d.; and ½ yard at 3s. 3d.= 1s. 7½d. Add 63s. and 5s. 3d. and 1s. 7½d. together=£3. 9s. 10½d.

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EXERCISES.

A GROCER'S BILL.

London, May 17, 1844.

Mr. Jones,

Bought of R. Taplin.

		d.		£	s.	d.
24 lb. of moist Sugar	at	9	per lb...			
18 lb. of Loaf Ditto	at	11	„	..		
7½ lb. of Currants	at	1/2½	„	..		
14 lb. of Raisins....................	at	10½	„	..		
6¼ lb. of Souchong Tea	at	5/8	„	..		
20 lb. of Rice......................	at	4	„	..		
				£4	19	3¾

A LINEN-DRAPER'S BILL.

Chatham, June 1844.

Mrs. Airey,

Bought of Henry Cooper.

		d.	£	s.	d.
Jan. 21.	11 yards of Sheeting ....at	2/4 per yard..			
,,	120 yards of broad Tape..at	7 ,, ..			
March 4.	A piece of Irish, 26½ yds...at	1/10 ,, ..			
,,	19½ yards of Flannel ....at	1/6½ ,, ..			
,,	3 doz. pair of Stockings..at	1/3½ per pr. ..			
April 14.	10¼ yards of Damask ....at	4/7 per yard..			
,,	4¾ ells of Diaper ......at	5/3 per ell ..			
			£14	12	8¾

A MERCER'S BILL.

Portsmouth, July 1844.

Mrs. Simpson,

Bought of Edward Peek.

	d.	£	s.	d.
10½ yards of Yorkshire Camblets ....at	11 per yard..			
13¼ yards of Velvet...............at	11/6 ,, ..			
17¼ yards of Silk ....at	4/3 ,, ..			
10¾ yards of Ditto ....at	5/7 ,, ..			
21 yards of flowered Silk ....at	6/4 ,, ..			
15½ yards of Persia ....at	3/5 ,, ..			
		£24	1	3½

A BUTCHER'S BILL.

London, May 1844.

. John Price, Esq.

To William Pettit.

			d.	£	s.	d.
Jan. 1.	Beef, Sirloin ...... 9½ lb. ..at	10 per lb...				
„ 4.	Veal .... 13 lb. ..at	9½ „	..			
„ 9.	2 Shoulders, Mutton 10¼ lb. ..at	8 „	..			
„ 14.	Beef, Boiling ...... 8 lb. 4 oz. at	8½ „	..			
„ 18.	Saddle of Mutton . 11 lb. 7 oz. at	9 „	..			
„ 20.	Veal ....14 lb. 4 oz. at	11½ „	..			
„ 23.	2 Legs of Mutton..13 lb. 11 oz. at	10 „	..			

£3 4 8½

A CORN-CHANDLER'S BILL.

Rochester, May 1844.

Mr. Titchmarsh,

To James Day.

		d.	£	s.	d.
43 Bushels of Wheat.............at 5/7 per bushel..					
13 „ of Barley.............at 4/9 „	..				
119 pecks of Beans...............at 1/7 per peck ..					
87 „ of Rye ....at 1/5 „	..				
20 bushels of Oats...............at 3/7 per bushel..					
64 „ of Peas...............at 1/3 .„	..				

£38 5 2

A TOBACCONIST'S BILL.

London, April 14, 1844.

Frederick Ellis, Esq.

To James Oliphant. .

Cwt. qr. lb.		d.		£	s.	d.
1 . 2 . 7 of Cut Tobacco ....,at	1/9 per lb...				
2 . 1 . 15 of Spanish Ditto..........at	1/10	,,	..		
0 . 3 . 10 of Ditto Ditto..........at	2/1	,,	..		
3 . 0 . 22 of Virginia Ditto ....at	1/9	,,	..		
1 . 2 . 5 of Ditto Ditto ....at	2/0	,,	..		
3 . 1 . 18 of Maryland Ditto ......at	1/5	,,	..		
				£125	5	3

CALCULATIONS AFTER THE MANNER OF

COMPOUND DIVISION.

6. To find the value of one article, the price of a dozen being given.

RULE.—Take the price in shillings as pence, and for every 3d. add one farthing to the value.

Note.—This Rule is the reverse of Rule the 1st.

Example 1.—If 12 books were sold for 15s., what is the price of each ?—The 15s., taken as 15d., the price of 1 book is 1s. 3d.

Example 2.—12 lb. of Tobacco cost 27s., how much per lb. ?—27s. taken as 27d.=2s. 3d., the price per lb.

Example 3.—A dozen fowls were sold for 43s. 9d. ; required the price of each.—For 43s. take 43d.=3s. 7d. ; for the 9 pence take 3 farthings ; which together=3s. 7¾d., the price of each.

1. Candles at 9s. per dozen, how much per lb. ?

2. A dozen Baskets cost 14s., how much was each ?

3. Eggs, at 2s. per dozen, what is that for each ?

4. Suppose 12 yards cost 17s. 6d., how much per yard ?

5. 12 days' expences amounted to 25s. 9d., how much per day ?

6. 1 doz. Silk Handkerchiefs came to 60s. 6d., how much a-piece ?

7. How much is Soap per lb., if 12 lb. cost 9s. 3d. ?

8. 12 Books cost 58s. 6d., what was the price of each ?

9. A dozen yards of Calico cost 27s. 9d., how much per yard ?

10. 12 lb. of Mutton came to 8s. 3d., what was charged per lb. ?

11. If a dozen Inkstands cost 18s. 6d., what is the price of each ?

12. A dozen lb. of Tea came to 72s. 9d., required the charge per lb. ?

7. Any number of dozens being given, to find the price of one article.

RULE.—Divide the Shillings in the price by the number of dozens, and call the quotient pence.

Example 1.—Suppose 36 lb. of Butter cost £1. 7s., how much per lb.? £1. 7s.=27s. and 27÷3=9s., taken as pence, 9d.=9d. per lb.

Example 2.—8 dozen of Wine Glasses cost £6. 8s.; required the price of each.

£6. 8s.=128s., and 128÷8=16, taken as pence, 16d.=16d. each.

Example 3.—If 120 lb. of Currants cost 55s., how much per lb. ?
55÷10=5½, taken as pence=5½d. per lb.

13. 24 lb. of Meat came to 18s.; how much per lb. ?

14. If 6 dozen boxes cost £1. 4s., how much was each ?

15. 5 dozen old Books cost £10. 15s., what was the average price ?

16. How much per lb., if 48 lb. of Tobacco sold for £6. 4s. ?

17. 9 dozen Snuff Boxes amount to 47s. 3d., what is the price of each ?

18. Paid £23. 8s. for 12 dozen Fowls, what is the average price of each ?

APPLICATION OF COMPOUND DIVISION IN THE CALCULATION OF THE SUBJOINED BILLS.

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A GROCER'S BILL.

London, May 1844.

Mr. Ivitt,

Bought of Charles Young.

			£	s.	d.
Feb.	8.	9 lb. of Loaf Sugar	0	7	10½
„	10.	15 lb. of Tea	4	3	9
„	13.	11 lb. of Coffee	1	8	5
„	17.	23 lb. of moist Sugar....................	0	16	3½
Mar.	3.	17¼ lb. of Raisins	0	19	8¼
„	5.	A box of Muscatels, weighing 1 qr. 17 lb.	2	18	1½
			£10	14	1¾

EXPLANATIONS.

The calculation of this bill differs from the preceding bills, inasmuch as the price per lb. is not specified. To ascertain the price per lb., the Rule for Compound Division is usually applied. It will be observed, that Concise Rules for mental calculation cannot be given as for Compound Multiplication. Nos. 6 and 7 may sometimes be used; but in general, Compound Division must be applied.

9 lb. of Sugar for 7s. 10½d., is 10½d per lb.

$$7s. \ 10\tfrac{1}{2}d. = 94\tfrac{1}{2}d.$$
$$9) \ \overline{}$$
$$10\tfrac{1}{2}d.$$

15 lb. of Tea for £4. 3s. 9d., is 5s. 7d. per lb.

$$£4. \ 3s. \ 9d. = 83s. \ 9d.$$
$$15) \ \overline{}$$
$$5s. \ 7d.$$

11 lb. of Coffee for £1. 8s. 5d., is 2s. 7d. per lb.

$$£1. 8s. 5d. = 28s. 5d.$$
$$11) \overline{}$$
$$2s. 7d.$$

23 lb. of Sugar for 16s. 3½d., is 8½d. per lb.

$$16s. 3½d. = 195½d.$$
$$23) \overline{}$$
$$8½d.$$

17½ lb. of Raisins for 19s. 8½d., is 1s. 1½d. per lb.

19s. 8½d. = 236½d. = 945 qrs., and 17½ lb. = 70 qrs.

$$ 94,5$$
$$7,0 \overline{}$$
$$13½d.$$

1 qr. 17 lb. for £2. 18s. 1½d., is 1s. 3½d. per lb.

£2. 18s. 1½d. = 58s. 1½d., and 1 qr. 17 lb. = 45 lb.

$$58s. 1½d.$$
$$45) \overline{}$$
$$1s. 3½d.$$

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A BUTCHER'S BILL.

Holloway, February 1844.

Rev. Peter Jones,

Bought of Edward Piper.

				£	s.	d.
Jan.	7.	Mutton, Leg and Shoulder..weight	17½ lb.....	0	13	10½
„	10.	Beef, Sirloin	„ 15 lb. 6 oz..	0	14	1
„	14.	Veal, Fillet	„ 8¼ lb.....	0	7	2½
„	16.	Beef, boiling.............	„ 14 lb. 10 oz.	0	9	10
„	21.	Lamb, Fore Quarter........	„ 11 lb. 7 oz..	0	10	6½
„	26.	Mutton, Saddle............	„ 9¾ lb.....	0	7	4
		Steaks ....,..	„ 3 lb. 14 oz..	0	3	6
				£3	6	4

17¼ lb. of Mutton for 13s. 10¾d., is 9¼d. per lb.

13s. 10¾d.=166½d.=665 qrs., and 17½ lb.=70 qrs.

$$7,0) \quad \frac{66,5}{9\frac{1}{4}}$$

15 lb. 6 oz. of Beef for 14s. 1d., is 11d. per lb.

14s. 1d. = 169d. = 1352 eighths of a penny.

And 15 lb. 6 oz. = 15$\frac{4}{12}$ = 15$\frac{3}{8}$ = 123 eighths of a lb.

$$123) \, 1352 \, (11d.$$

$$\frac{122}{\cdots} \quad \text{Say 11d. per lb.}$$

Note.—The divisor and dividend must always be brought into the same name. In this instance, they are brought into eighths.

8¼ lb. of Veal for 7s. 2¼d., is 10¼d. per lb.

7s. 2¼d. = 86¼d. = 346 qrs., and 8¼ lb. = 33 qrs.

$$33) \, 346 \, (10\frac{1}{4}d. \text{ nearly.}$$

$$\frac{16}{}$$

14 lb. 10 oz. of Beef for 9s. 10d., is 8d. per lb.

9s. 10d.=118d.=944 eighths of a penny.

14 lb. 10 oz.= 14$\frac{10}{16}$=14$\frac{5}{8}$ lb.=117 eighths of a lb.

$$117) \, 944 \, (8d.$$

$$\frac{8}{}$$

11 lb. 7 oz. of Lamb for 10s. 6¼d., is 11d. per lb.

10s. 6¼d.=126¼d.=505 qrs. of a penny.

11 lb. 7 oz. may be taken as 11½ lb.= 46 qrs. of a lb.

$$46) \, 505 \, (11d. \text{ very nearly.}$$

$$\frac{45}{}$$

9¾ lb. of Mutton for 7s. 4d., is 9d. per lb.

7s. 4d.=88d.=352 qrs. of a penny.

9¾ lb.= 39 qrs. of a lb.

39) 352 (9d.

—————

..1

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3 lb. 14 oz. of Steaks for 3s. 6d., is 11d. per lb.

3s. 6d.=42d.=336 eighths of a penny.

3 lb. 14 oz.=3¹⁵⁄₁₆ lb.= 3⅞ = 31 eighths of a lb.

31) 336 (11d. nearly.

—————

26

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As 26=31, nearly;—the price is more than 10¾d. per lb., and something less than 11d.

•   ————

A CHEESEMONGER'S BILL.

St. Albans, May 1844.

Mr. Stephen Morris,

Bought of Samuel Eddis.

			£	s.	d.
April	7.	17 lb. of fresh Butter...................	1	3	4½
		5¼ lb. of Cheese.......................	0	4	9¾
„	14.	11¼ lb. of Bacon	0	9	9¾
„	15.	13 lb. of Gloucester Cheese..............	1	0	7
„	21.	A fine Ham, weighing 12¼ lb..............	1	2	11
May	2.	3 Cheshire Cheeses, weighing 127 lb........	7	8	2
			£11	9	8

Find the price of 1 lb. of the respective quantities.

A LINEN-DRAPER'S BILL.

Manchester, June 1844.

Joseph Arrowsmith, Esq.

Bought of W. & F. Jones.

	£	s.	d.
40 ells of Holland	9	3	4
37 yards of Cambric	20	13	2
27 ells of Dowlas	1	12	7¼
43 yards of Linen	2	17	4
16¼ yards of Cotton	1	5	4
25¼ yards of Muslin	7	2	4¼
19¼ yards of Flannel.............................	1	17	4¼
	£44	11	6¼

Find the price of each ell, yard, &c.

A GROCER'S BILL.

Liverpool, June 1844.

Thomas Bishop, Esq.

Bought of Ellis, Tucker, & Co.

			£	s.	d.
Jan.	8.	10 lb. of Souchong Tea................	3	6	8
	10.	39 lb. of Rice.......................		14	7¼
	„	12 Cakes of Chocolate	1	18	0
	19.	14¼ lb. of Congou Tea	3	6	6
	28.	22 lb. of Loaf Sugar....................	1	5	8
Feb.	2.	24¼ lb. of moist Sugar		17	4¼
Mar.	5.	15¼ lb. of Coffee........................	1	15	5¼
			£13	4	3¼

Find the price of each lb. &c.

8. To find the value of goods per Cwt. at any price per lb.

As 112 lb. (1 Cwt.) at 1d. per lb.=112d.=9s. 4d.

RULE.—Multiply 9s. 4d. by the pence given: if the price include farthings, add 2s. 4d. to the price for every farthing.

Example 1.—1 Cwt. of Soap, at 7d. per lb.

9s. 4d. multiplied by 7=65s. 4d.=£3. 5s. 4d. per Cwt.

Example 2.—1 Cwt. of Iron, at 3¾d. per lb.

	9s.	4d.
multiplied by		3
	28	0
add 3 times 2s. 4d. =	7	0
	35s.	0d.=£1. 15s. per Cwt.

EXERCISES.

19. 1 Cwt. of old Lead, at 1½d. per lb.

20. 1 Cwt. of Copper, at 3¼d. per lb.

21. 1 Cwt. of Sugar, at 5d. per lb.

22. 1 Cwt. of Soap, at 7¼d. per lb.

23. 1 Cwt. of Cheese, at 9¼d. per lb.

24. 1 Cwt. of Tin, at 11d. per lb.

25. 3 Cwt. of Salt, fine, at 1¼d. per lb.

TO CALCULATE INTEREST.

AT £5 PER CENT FOR MONTHS.

The Interest at £5 per Cent. is one penny per £1 for every month.

RULE.—Call the given pounds—pence, and multiply by the number of months.

Example 1.—Required the Interest on £150 for 7 months.

  £150, taken as pence=12s. 6d. for 1 month.

    multiply by   7 months.

        £4. 7s. 6d. is the Interest.

Example. 2.—What is the Interest on £430. 5s. for 4 months?

  £430. 5s. called pence=35s. 10¼d. for 1 month.

    multiply by   4 months.

       £7. 3s. 5d. Interest.

OBSERVE—One farthing must be added for every 5s. Nothing under 5s. gives a farthing for Interest.

AT £5 PER CENT. FOR DAYS.

RULE.—Multiply either the pounds by one-third of the days, or the days by one-third of the pounds (as most convenient); reject the units figure, and the answer will be in pence, nearly.

Example 1.—Required the Interest on £78 for 45 days.

      £78

  multiply by   15 (⅓ of 45 days.)

     1 17|0 reject 0, and 117d.=9s. 7d. nearly.

NOTE.—The result will not be exact, because by this method 360, instead of 365 days, are computed to the year.

To ascertain exactly, add 1/72d part.

Example 2.—What is the Interest on £150 for 29 days.

      29 days

  multiply by 50 (⅓ of £150.)

     145|0=145d.=12s. 1d. nearly.

At £2½ per Cent., calculate as above, and divide by.... 2

 £1¼ ....,...... 4

 £10 per Cent, calculate as above, and multiply by.. 2

 £20 4

EXERCISES.

1. What is the Interest on £80 for 2 months, at £5 per Cent. ?

2. What is the Interest on £120 for 5 months, at £5 per Cent. ?

3. Required the Interest on £200 for 35 days, at £5 per Cent.

4. Required the Interest on £175 for 21 days, at £5 per Cent.

5. How much is the Interest on £370. 15s. for 9 months, at £5 per Cent. ?

6. Ascertain the Interest on £250 for 27 days, at £5 per Cent.

7. On £500 at £2½ per Cent., what is the Interest for 8 months ?

8. On £315. 10s. at £2½ per Cent., what is the Interest for 24 days ?

9. Find the Interest on £300 for 2 months, at 10 per Cent.

10. Find the Interest on £428 for 25 days, at £20 per Cent.

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AT £6 PER CENT. FOR MONTHS.

RULE.—Multiply the pounds by the months, reject the unit's figure, and the answer is in shillings.

Example 1.—What is the Interest on £320 for 8 months?

           £320

multiply by   8 months.

     256|0 (0 rejected)=256s.=£12. 16s.

Example 2.—Required the Interest on £47. 15s. for 9 months.

           £47. 15s.

multiply by   9 months.

       £429. 15s.=£429¾.

reject 9, unit's figure, and also the fraction=42s.=£2. 2s.

If the rejected figure be multiplied by 1¼ the result will be in pence, which added to the pounds and shillings gives £2. 2s. 11d. for the Interest.

ANOTHER RULE.—Calculate as 5 per Cent., and add ⅕.

At £6 per Cent. for Days.

RULE.—Multiply the Interest of 1 month by the days given, divide by 30, and the quotient will be the Interest nearly.

At £3   per Cent., calculate as above, and divide by....2

£2 3

£1½ 4

£12   per Cent., calculate as above, and multiply by ..2

         &c. &c. &c.

Exercises.

11. How much is the Interest on £40 for 3 months, at £6 per Cent. ?

12. How much is the Interest on £75 for 7 months, at £6 per Cent. ?

13. Find the Interest on £205 for 9 months, at £6 per Cent.

14. Find the Interest on £450 for 11 months, at £6 per Cent.

15. Required the Interest of £87. 10s. for 22 days, at £6 per Cent.

16. Required the Interest of £584. 15s. for 17 days, at £6 per Cent.

17. What is the Interest of £310 for 5 months, at £3 per Cent. ?

18. What is the Interest of £286. 10s. for 21 days, at £3 per Cent. ?

19. At £1½ per Cent., what is the Interest on £175 for 8 months ?

20. At £12 per Cent., what is the Interest on £400 for 23 days ?

At any Rate per Cent. for One Year.

RULE.—Multiply by double the Rate, reject the unit's figure, and you have the Interest in shillings.

Example 1.—What is the Interest on £575 for 1 year, at £3½ per Cent. ?

     £575

       7 halves=3½

     ———

402|5, reject 5 = 402s. = £20. 2s. (interest) and for 5

           rejected add 6d. = £20. 2s. 6d.

             I 2

Example 2.—Required the Interest of £1243 for 1 year, at £4½ per Cent.

> £1243
>   9 halves = 4½
> ——————
> 1118|7 = 1118s. = £55. 18s. 8d. Interest.

To find the Interest for months and days, aliquot parts must be taken, as shewn in the RULE OF INTEREST.

EXERCISES.

21. Required the Interest on £220 for a year, at £3½ per Cent.
22. At £3 per Cent., required the Interest of £543 for a year.
23. What is the Interest on £186. 17s. at £4 per Cent., for a year?
24. Find the Interest of £754 for a year, at £4½ per Cent.
25. How much is the Interest of £1728 in the 3½ per Cents. for a year?
26. On £640 what is the yearly Interest at £2½ per Cent.?
27. At £4 per Cent., what is the Interest of £860 for 2 years?
28. Required the Interest of £2065 for half a year, at £3¼ per Cent.

In large mercantile establishments, banking-houses, &c. Interest Tables are referred to.

For the ready dispatch of business the Rules given above may be found useful.

⁓⁓⁓⁓⁓⁓⁓⁓⁓⁓

IN CALCULATING DISCOUNT, PROFIT, LOSS, &c.

NOTE—That

> 1 in 20 is at the rate of 5 per Cent.
> 2 10   „
> 3 15   „
> 4 20   „
>   &c. &c. &c.

EXTENDED MULTIPLICATION TABLE.

	13	14	15	16	17	18	19	20
12	156	168	180	192	204	216	228	240
13	169	182	195	208	221	234	247	260
14		196	210	224	238	252	266	280
15			225	240	255	270	285	300
16				256	272	288	304	320
17					289	306	323	340
18						324	342	360
19							361	380
20								400

A SHORT METHOD TO REDUCE *Cwt. qrs. & lbs.* INTO *lbs.*

Example 1.—Bring 75 Cwt. 2 qrs. 9 lbs. into lbs.

First, 2 qrs. 9 lbs. = 65 lbs.

Put the Cwt. given before 65, in 1 line, thus

7565 to which add

75 Cwt. multiplied by 12 = 900

8465 lbs.

Example 2.—Change 147 Cwt. 3 qrs. 22 lbs. into lbs.

3 qrs. 22 lbs. = 84 lbs.+22 lbs = 106 lbs.

14806 to which add

147 Cwt. multiplied by 12= 1764

16570 lbs.

* NOTE.—If the number of lbs. exceed 100, as in this Example, the last figure of the Cwt. must be increased one more. Hence the 147 Cwt. is put as 148.

1. What part of a shilling is 1½d. ?

2. 6 lb. of Coffee, at 3s. 7d. per lb.

3. 100 yards of Tape, at 2d. per yard.

4. Salt, ½d. per lb., how much per Cwt.

5. 17 lb. of Beef, at 8½d. per lb.

6. A dozen of Wine at 3s. 6d. per bottle.

7. What are ¾ of a Pound sterling?

8. How many Sixpences in 23s. 6d. ?

9. 1¼ dozen of Eggs, at 1½d. each.

10. 9 ells of Diaper, at 18s. 4d. per ell.

11. 100 Pears at 1¼d. each.

12. A Leg of Mutton, weight 9 lb. 2 oz., at 7½d. per lb.

13. Beef weighing 8½ lb., came to 6s. 4½d., how much per lb. ?

14. 1 Cwt. of Soap, at 8d. per lb.

15. In £2. 10s. how many Half-crowns ?

16. 20 lb. of Stilton Cheese, at 14d. per lb.

17. A piece of Irish containing 27½ yards, at 3s. 4d. per yard.

18. A dozen pairs of Shoes, at 3s. 8d. per pair.

19. Butter, 13½ lb., at 10d. per lb.

20. 8 bushels of Pears, at 4s. 9d. per bushel.

21. 35 yards of Ribbon, at 4d. per yard.

22. 4 dozen Slates, at 8d. each.

23. Oranges, 5 for 2d., how many for half-a-crown ?

24. ¾ of a lb. of Tea, at 6s. 4d. per lb.

25. 105 Loaves, at 7d. per Loaf.

26. 2 dozen pair of Braces, at 14½d. per pair.

27. 3 Cwt. of Lead, at 2d. per lb.

28. 45 Apples, at a 1¼d. each.

29. 9 dozen Knives, at 18s. 7d. per dozen.

30. Gained 1s. 6d. by selling Tea per lb., how much is gained on 1 Cwt. ?

31. 7¼ dozen pairs of Gloves, at 2s. 7d. per pair.

32. What part of a £ is 16s. 8d. ?

33. Veal, weighing 11¾ lb., at 10½ per lb.

34. 100 yards of Silk, at 3s. 4d. per yard.

35. How much is ¾ of a Sovereign.

36. 100 Pears, at 6d. per dozen.

37. Rice, 22¾ lb. at 4d. per lb.

38. Tobacco, at 2s. 6d. per lb., what quantity may be had for 10 guineas ?

39. How much is one-fourth of 23s. 9d.

40. 31 yards of Silk, at 9s. per yard.

41. 87 Tons of old Iron, at 14s. per Ton.

42. From ¾ of a £ take ¼ of a Shilling.

43. 400 yards, at 3d. per yard.

44. 120 ounces, at 5s. 6d. per ounce.

45. Gooseberries, 4d. a quart, how many quarts for 10s. ?

46. How many Half-crowns in 50 Guineas ?

47. Paid 19s. 3d. for 9 lb. of Chocolate, how much per lb. ?

48. Pepper, 3½d. per ounce, how much per lb. ?

49. 4½ dozen Inkstands, at 8¼d. each.

50. Cloth at 2s. 1d. per yard, how much will 20 yards come to ?

51. 1 Cwt. of Loaf Sugar, at 9¼d. per lb.

52. How much will 2d. a day amount to in a year ?

53. A box of Raisins, weight 26¼ lb., at 15d. per lb.

54. 120 Baskets at 8d. each.

55. 10¼ lb. of boiling Beef, at 7½d. per lb.

56. What is ¼ of ¾ of a Pound ?

57. ¼ of a yard of Cloth, at 11s. 9d. per yard.

58. In £1. 15s. how many Farthings ?

59. 2 Legs of Pork, each 8 lb., at 11d. per lb.

60. Are ⅔ of a £ more or less than ¾ of a £—how much ?

61. Potatoes, at 1¼d. per lb., how much per Cwt. ?

62. If 6d. a day be given to the poor, how much in a year ?

63. 240 lb. of Copper, at 14d. per lb.

64. 7¼ yards at 4s. 6d. per yard.

65. How much are ⅞ of 15s. ?

66 ¼ Cwt. of Cheese, at 10d. per lb.

67. 3¼ dozen Apples, at ¾d. each.

68. 31 yards of Holland, at 10¼d. a yard.

69. 42 boxes, at 1s. 6d. per dozen.

70. Candles, 7s. 6d. per dozen lb. how much per lb. ?

71. A gross of Oranges, at 1s. 7d. per dozen.

72. What is the square of 9—of 12—of 15—of 20 ?
73. What is the cube of 3—of 8—of 12—of 20 ?
74. Wine, at 2s. 6d. per pint, what would 2 gallons amount to ?
75. 51 yards of Holland, at 9¼d. per yard.
76. 5 dozen of black Lead Pencils, at 5d. each.
77. 40 Silk Handkerchiefs, each 3s. 6d.
78. A Loin of Mutton, 6 lb. 6 oz., cost 5s. 4d., how much per lb. ?
79. Figs, £2. 2s. for ¼ of a Cwt., how much per lb. ?
80. 3 dozen yards of Cloth, at 1s. 1d. per yard.
81. How much is Treacle per Cwt., at 6¼d. per lb.
82. How many square yards in a Carpet 10 feet long and 8¼ feet wide ?
83. 28 days' Wages, at 5s. 6d. per day.
84. 360 Oranges at 8d. per dozen.
85. If 1½d. is gained on 1 shilling, what is gained on £5?
86. 7⅞ yards of Cloth, at a guinea per yard.
87. If 4s. 8¼d. and 5s. 3½d. be paid out of a Sovereign, what is the change ?
88. 100 Flower-pots, at 3d. each.
89. Lamb, 9 lb. 10 oz. at 11d. per lb.
90. If 3 Apples are sold for 1d., how much will 5 dozen come to ?
91. 17 lb. of Dorset Butter, at 10d. per lb.
92. 100 Eggs, at 1s. 6d. per dozen.
93. 15 yards of Cloth, at 19s. 11¼d. per yard.
94. How much will 1¼d. a day amount to in a year ?
95. Pay for 3 lb. of Beef, at 10¼d. per lb.; 4 lb. of Mutton, at 9d. per lb.; and what will remain out of 10s.?
96. A stone of Meat, at 9¼d. per lb.
97. Cheese, 9¼ lb., at 9¼d. per lb·
98. 25 lb. of moist Sugar, at 7d. per lb.
99. 53 yards of Baize, at 11d. per yard.
100. A Cwt. of Soap came to 65s. 4d., how much per lb. ?

FINIS.